Default Risk and the Pricing of U.S. Sovereign Bonds

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June 3, 2020

Abstract

We examine the relative pricing of nominal Treasury bonds and Treasury inflation-protected securities (TIPS) in the presence of United States default risk. Higher bond yields are associated with a higher U.S. credit default swap premium, but more so for TIPS. This leads to a narrower breakeven inflation (BEI). An estimated no-arbitrage model shows BEI is related to differing expectations of loss given default on the two Treasury securities and that most of the relative *mispricing* after the crisis can be attributed to default risk. Our finding suggests credit risk is embedded in the pricing of U.S. sovereign debt.

JEL classification: E4, E6, G12.

Keywords: Treasury, TIPS, Breakeven Inflation, Default Risk, Loss Given Default.

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[¶]We would like to thank seminar participants at the 2019 AFA Annual Meeting, the Banco Central de Argentina, the 2019 BI-SHoF Conference, Baylor University, the Deutsche Bundesbank, the Federal Reserve Bank of Atlanta, the Federal Reserve Bank of Chicago, Georgia Institute of Technology, the 2019 HEC-McGill Winter Finance Workshop, the Sixth International Conference on Sovereign Bond Markets, the 2019 SFS Cavalcade, the 2019 South Carolina FIFI Conference, the 2019 UBC Winter Finance Conference, the 2019 SoFiE Annual conference, the University of Massachusetts, the 2019 European Finance Association, and the 2019 Northern Finance Association for helpful comments. Thanks to Patrick Augustin, Geert Bekaert, Mikhail Chernov, Alexander David, Stefania D'Amico, Matthias Fleckenstein, Robin Greenwood, Espen Henriksen, Thomas King, Leonid Kogan, Lukas Schmid, and Luis Viceira for useful suggestions. The usual disclaimer applies.

1 Introduction

Does United States Treasury default risk have the same impact on the pricing of all U.S. Treasury obligations? We investigate this question through the lens of the relative pricing of nominal and inflation protected Treasury securities. Our interest is motivated by the pairwise mispricing between nominal and real bonds documented in Fleckenstein, Longstaff and Lustig (2014). The authors show that a strategy replicating inflation-protected securities through inflation swaps, STRIPs, and nominal Treasuries generates large and persistent arbitrage profits. Their empirical analysis suggests that much of the profitability of the strategy is likely to be explained by slow-moving capital that prevented the profits from being arbitraged away. Our study asks if part of this differential might be accounted for by differences in exposures to default risk present in nominal and inflation-protected (TIPS) Treasury securities.

Our investigation of the role of default risk in this pricing differential may be surprising given the frequent treatment of Treasury obligations as default risk-free. However, the financial crisis of 2008-2009 and its aftermath have suggested that this perception may need to change. Chernov, Schmid and Schneider (2019) note that the premium paid to insure U.S. sovereign debt as measured by credit default swap (CDS) spreads rose to nearly 100 basis points during the crisis, and remained elevated since. The authors show that a macro-finance model with a non-trivial probability of sovereign default can replicate this pattern in the United States and other developed markets. Repeated political conflict over the debt ceiling in the United States in 2011, 2013, and 2017 has also contributed to questions about the risk-free status of U.S. sovereign debt. The 2011 conflict led to Standard and Poor's downgrading the status of the United States Treasury as an obligor from AAA to AA+.

Even if a default is possible, why might we see a resulting differential in the pricing of nominal and inflation-protected debt? History suggests that there is considerable uncertainty as to how forms of debt might be treated in the case of a sovereign default. As discussed in Duffie, Pedersen and Singleton (2003), sovereign defaults rarely play out in the way modeled in structural or reduced form models of corporate credit risk. Rather than a single event, a sovereign entity weighs the costs and benefits of continuing to pay its obligations against the reputational cost of default. When default occurs, it is more likely that the debt will be restructured or renegotiated than that an outright liquidation will take place. This renegotiation involves a considerable amount of uncertainty. Duffie, Pedersen and Singleton (2003) examine the case of the Russian default on its ruble-denominated debt in 1998, and its impact on dollar-denominated MinFins. The authors show that uncertainty around the treatment of this debt, which was considered domestic, relative to foreign Eurobonds had a large impact on the relative pricing of the obligations. Similarly, Zettelmeyer, Trebesch and Gulati (2019) examine the variation in ultimate recovery in present value terms of holders of Greek debt during the 2012 restructuring. All bondholders were provided the same package of securities, which implied large differences in present value loss given default across holders of different bonds.[1]

With this background in mind, our empirical work examines the question of whether the spread between like-maturity inflation swaps (ILS) and breakeven inflation (BEI), the difference between yields of nominal and inflation-protected U.S. Treasuries, is correlated with default risk. Using spreads on CDS written on U.S. Treasury obligations to proxy for overall risk of default, we find a statistically and economically significant relation between the ILS-BEI spread and CDS spreads. Specifically, over the period 2008 through 2015, a one standard deviation increase in the CDS spread (16 basis points) is associated with a 3.1 basis point increase in the hedged breakeven inflation, about 10% of the average ILS-BEI spread throughout our sample.[2] This relation is not simply a manifestation of dislocation during the financial crisis; the result holds in the subsample from 2010 onward. In fact, the statistical significance of coefficient estimates on CDS spreads increases in the post-crisis sample. Furthermore, we demonstrate that the relation is robust to controls for liquidity and slow-moving capital. Lastly, we extend our empirical tests using the U.K. data to verify our hypothesis. Consistent with our U.S. results, the U.K. ILS-BEI spread loads positively and significantly on U.K. CDS spreads after the crisis.

In order to better understand the source of this covariation, we derive a new affine model of defaultable nominal and inflation-protected sovereign debt building on Monfort et al. (2020). In this model, a sovereign entity issues multiple bonds with differences in their possible losses given default. We view this modeling approach as a convenient way to capture uncertainty surrounding the treatment of different bonds in case of default, such as the renegotiation of Russian and Greek bonds discussed above. In our context, this uncertainty applies to the treatment of U.S. sovereign bonds in case of a default. The specific uncertainty that we have in mind relates to whether a default event would trigger a default on all U.S. Treasury obligations and, if so, the ultimate recovery of present value of the bonds under consideration.[3] For example, concerns about whether the inflation

[1]In Figure 5 of Zettelmeyer, Trebesch and Gulati (2019) titled "Bond-by-bond haircuts, by remaining duration," the authors show significant heterogeneity across maturities in haircuts suffered by holders of nominal Greek debt, ranging from 20% to 90%.

[2]Hedged breakeven inflation is defined as the spread between the inflation-linked swap rate and the Treasury-based breakeven inflation rate of the same maturity (ILS-BEI).

[3]The law is not clear on whether cross-default provisions apply to U.S. Treasury debt. According to Schwarz (2014), "It is unclear whether any U.S. debt securities contain cross-default clauses. The statute setting forth procedures for the U.S. government to issue debt securities makes no mention of these types of

indexing of TIPS would be removed upon default would generate uncertainty about whether TIPS investors would effectively suffer deeper losses of present value than nominal Treasury bond holders. The closed-form pricing formulae in our model explicitly relate the spread between inflation-linked swaps and the breakeven inflation rate implied by the prices of nominal and inflation-indexed securities to differences in loss given default.

We estimate the parameters of the model with the extended Kalman filter, targeting the term structures of overnight indexed swaps (OIS), inflation-linked swaps, nominal Treasuries, TIPS, and CDS. For each of these five term structures, we fit five maturities between one and ten years. Additionally, our estimation includes the inflation and a proxy for TIPS liquidity, for a total of 27 time series. The model is characterized by six factors: three nominal and real term structure factors, two credit factors, and one liquidity factor. Our results indicate that the model is able to simultaneously capture most of the variation in all observable variables, showing outstanding fitting performance for a relatively low number of factors, with most R^2 measures exceeding 90%. A decomposition of the spreads between ILS and Treasury breakevens and show that credit risk factors are able to capture between approximately 50% and 100% of the total ILS-BEI variation during our sample period. The remaining variation is explained by illiquidity issues in the TIPS market, specifically at the height of the crisis in 2008-2009. The results support our conjecture that nominal and inflation-protected treasuries are affected differently by sovereign default risk.

Our paper contributes to at least three broad strands of the fixed income literature. The first area to which we contribute is the relative pricing of nominal and inflation-protected securities. This literature seeks to extract information about inflation risk premia, inflation or deflation expectations, and mispricing from Treasury prices.[4] Most closely related to our analysis, Fleckenstein, Longstaff and Lustig (2014) document apparent no-arbitrage violations in the pricing of nominal and inflation-protected securities, and conclude that the arbitrage arises due to slow-moving capital. Their arbitrage measure is closely related to the hedged ILS-BEI spread used in our empirical analysis. We differ from this and other papers in the literature in explicitly considering the impact of default risk on the relative pricing of nominal and inflation-protected securities. Our results suggest that part of the pricing differential is related to credit risk.[5]

clauses ..."

[4] Buraschi and Jiltsov (2005), Ang, Bekaert and Wei (2008), Hordahl and Tristani (2012), Chernov and Mueller (2012), Haubrich, Pennacchi and Ritchken (2012), and Roussellet (2019) examine reduced-form affine pricing models for the purpose of extracting information about inflation risk premia. Christensen, Lopez and Rudebusch (2012) and Fleckenstein, Longstaff and Lustig (2017) extract deflation probabilities from real Treasuries.

[5] Fleckenstein, Longstaff and Lustig (2014) note that inflation-protected securities are not necessarily default risk-free, but suggest that since CDS do not distinguish between nominal and inflation-protected

A second strand of literature investigates the role that liquidity risk plays in driving the difference in TIPS and nominal Treasury prices.[6]. Pflueger and Viceira (2016) suggest that there is a large and economically significant liquidity premium that affects the relative pricing of nominal and real bonds in both the U.S. and the U.K, and that this liquidity premium is largely captured by the ILS-BEI that is the focus of our empirical work. Christensen and Gillan (2018) and Moench and Vladu (2018) also use the ILS-BEI spread as a proxy for liquidity risk because high liquidity is attributed to the swap market in the U.S. Our analysis suggests that the ILS-BEI differential reflects not just a liquidity risk premium, but also a credit risk premium.

The third strand of literature investigates the role of default risk in the pricing of sovereign securities and CDS. Chernov, Schmid and Schneider (2019) use the rise in CDS premia in the U.S. and developed countries after the crisis to motivate a macrofinance model in which CDS premia reflect default probabilities.[7] Their model is able to generate the high premium paid to insure U.S. sovereign debt. The authors' framework contrasts with friction-based explanations for CDS premia such as counterparty risk as in Siriwardane (2019) or financial regulation as in Klingler and Lando (2018).[8]. Our paper similarly considers U.S. default with non-trivial probability but focuses on the effect of default risk on the relative pricing of different securities issued by a sovereign entity.

debt, default risk is unlikely to explain the arbitrage profits. See also Simon (2015) for a related analysis in the Euro-Area context.

[6]Grishchenko and Huang (2012) construct inflation risk premia employing only TIPS yields and control for the liquidity premium between TIPS and nominal bonds. Abrahams, Adrian, Crump, Moench and Yu (2016) decompose real and nominal yields into liquidity, inflation, and real interest rate risk components in an affine term structure model and conclude that forward breakeven inflation is primarily driven by risk and liquidity premia. D'Amico, Kim and Wei (2018) propose a substantial liquidity premium as the primary factor driving the wedge between TIPS yields and real risk-free rates, thus causing distortions in the term structure of breakeven inflation. Andreasen, Christensen and Ridell (2017) identify liquidity risk in TIPS with the average deviation across bonds from the predictions of a no-arbitrage pricing model. Finally, Fleckenstein and Longstaff (2019) can be interpreted to suggest that the ILS-BEI could be related to intermediary balance sheet constraints.

[7]Related work by Ang and Longstaff (2013) estimates an affine multi-factor model of U.S. and European state and country credit default swaps and concludes that systemic sovereign risk is strongly linked to financial market variables. Augustin and Tédongap (2016) provide an extended analysis for a cross-section of 38 different countries.

[8]Arora, Gandhi and Longstaff (2012) show that counterparty risk is priced in the CDS market using data covering the height of the 2008 crisis, but the magnitude is trivial because of the full collateralization of CDS liabilities. A summary of potential drivers of CDS spreads is also provided in survey of Augustin (2018*a*) and the references therein.

2 The Case of a U.S. Default

Our empirical analysis crucially relies on four notable financial instruments, namely credit default swaps on the U.S. government (CDS), sovereign U.S. nominal and inflation-indexed bonds, and inflation-indexed swaps. This Section details the important institutional features of these market instruments.

2.1 Nominal treasuries and TIPS

The main two instruments of debt issuance for the U.S. government are cash-denominated Treasuries (nominal) and *Treasury inflation-protected securities* (TIPS). Nominal zero-coupon bonds pay their nominal face value to the bond-holder at maturity. In contrast, zero-coupon TIPS holders earn the inflation-adjusted face value of the bond at maturity. Since cumulative inflation tends to be positive, TIPS tend to trade at a premium compared to nominal bonds. For both nominal bonds and TIPS, yields at issuance are determined through an auction process involving numerous market participants. According to Treasury Direct, as of April, 2020, the total principal value of Treasury securities outstanding is $18,104 billion, of which $1,493 billion, or 8% are TIPS. The dollar amount of TIPS outstanding is comparable in magnitude to each of the respective markets for asset-backed securities, federal agency securities, and U.S. money market instruments.[9]

The TIPS inflation adjustment is computed using the *seasonally non-adjusted consumer price index* for all urban consumers in the U.S. (CPI-U). CPI data is published monthly by the Bureau of Labor Statistics with a lag of about one and a half months, making the realized inflation unavailable when TIPS mature. TIPS payments thus include an *indexation lag* — the index used to determine their cashflows is a linear interpolation of CPI-U observed between two and three months before. The inflation-adjusted principal paid back at maturity is calculated by multiplying the face value of the bond by the cumulative *index ratio*. TIPS embed a *deflation floor*, such that they return the full face value even if cumulative inflation realized over the bond lifetime is negative.[10]

Despite the indexation lag, it would be difficult for the U.S. government to inflate away outstanding TIPS. Technically, it would be possible for the sovereign to resort to seignoriage to pay back maturing TIPS and current coupon payments without realizing the consequence of increased

[9] https://www.sifma.org/resources/research/fixed-income-chart/

[10] We consider zero coupon bonds in this study. Note however that most of nominal bonds and TIPS issued by the U.S. sovereign are coupon bonds paying on a semi-annual basis, but TIPS are only issued in terms of five, ten, twenty, or thirty years. For TIPS coupon payments, the coupon rate is fixed and paid on the inflation adjusted principal. For coupon payments, there is no deflation floor and the inflation-adjustment is computed using the index ratio realized over the last 6 months.

inflation. However, the inflation adjustment will materially impact any remaining outstanding TIPS, increasing the future interest payments of the government. Should the U.S. government refuse to honor the TIPS indexation, this would likely trigger a credit event and force the payoff of U.S. CDS contracts (see below). In case of default, nominal bonds and TIPS have the same level of seniority.

Leaving aside the embedded deflation floor, TIPS can be theoretically replicated by combining nominal bonds and inflation-linked swaps (ILS), as shown in Fleckenstein, Longstaff and Lustig (2014). ILS allow for the buyer to earn cumulative inflation in exchange for a fixed rate, relative to the notional agreed upon at inception. Inflation swaps are costless to write, and they are typically zero-coupon. As of April 2012, the average daily brokered inflation swap activity was estimated to be \$350 million, concentrated around the 10-year maturity. Importantly, despite a low trading frequency averaging about 2.2 contracts per day, the market for inflation swaps appears fairly liquid, with bid-ask spreads from proprietary data averaging below 3 basis points.[11] Keeping with the standard for swap contracts, ILS are collateralized, thus subject to minimal counterparty risk. In the remainder of the paper, we will assume that ILS are virtually risk-free.

In a frictionless economy, for a given maturity n, no arbitrage implies that the zero-coupon ILS rate is equal to the spread between the nominal and TIPS zero-coupon yields, called *breakeven inflation rate* (BEI):

$$\mathrm{ILS}_t^{(n)} = R_t^{(n)} - R_t^{*(n)} = \mathrm{BEI}_t^{(n)} . \tag{1}$$

This measure is the zero-coupon equivalent of Fleckenstein, Longstaff and Lustig (2014), who show that the cash flows of any traded nominal Treasury bond can be replicated by a portfolio of TIPS, U.S. Treasury STRIPS, and inflation swaps. We equivalently call this spread ILS-BEI, *mispricing*, or *hedged breakeven*.

In practice, researchers have observed large deviations from this no-arbitrage relationship over the maturity spectrum. Figures I and II present the five years to maturity series of ILS and BEI and the term structure of the spread between inflation swap rates and zero-coupon BEI, respectively. These deviations from the no-arbitrage relationship are quite persistent, and average between 30 and 36 basis points depending on the maturity. In the midst of the crisis, they reached more than 200 basis points.

Most of this apparent mispricing has been previously attributed to the low liquidity of TIPS relative to nominal bonds and ILS or to slow-moving capital (see e.g. D'Amico, Kim and Wei (2018) or Fleckenstein, Longstaff and Lustig (2014)). Campbell, Shiller and Viceira (2009) suggest the

[11] https://libertystreeteconomics.newyorkfed.org/2013/04/how-liquid-is-the-inflation-swap-market.html and *JPMorgan Investment Insight: Inflation Derivatives.*

premium is related to the cost of supplying inflation protection and is typical under normal market conditions. Inflation swaps, Treasuries, and TIPS all trade over-the-counter and may be subject to varying liquidity risk or counterparty credit risk in the case of ILS. We argue that the ILS-BEI spread is also significantly related to the risk of default of the U.S. sovereign that we associate with U.S. CDS. In our analysis, we control for all these potential confounding factors in the analysis, and abstract away from the embedded deflation floor in TIPS and the tax-related issues. We note that the deflation floor drives the price of TIPS upward, making the observed TIPS yield lower than the one used in the no-arbitrage argument. This would lead us to *underestimate* the ILS-BEI spread, thus the size of the potential mispricing.

2.2 U.S. Sovereign CDS

Credit default swaps (CDS) are OTC instruments designed to protect bond investors from a contingent *credit event* of the issuing entity. In practice, a bond investor (*protection buyer*) entering a CDS agrees to pay a fixed premium, typically called the *CDS spread*, on a regular basis to the *protection seller*, her counterparty. In case of a credit event, the contract terminates and the seller has to deliver the *loss given default* (LGD) realized on the bond to the buyer, making her earn the entire face value of the bond upon default. As is standard for swap contracts, the premium is indexed on a notional amount agreed upon at inception and is set such that the original cost of issuance is zero. While not free from counterparty credit risk, CDS are typically collateralized.

The International Swaps and Derivatives Association (ISDA) provides legal details that define the triggers for the termination of CDS, which type of obligations are considered, and how the LGD and repayment operates depending on the underlying bond issuer (see ISDA (2003, 2014)). In the case of the United States Treasury, a credit event is observed whenever the government either *(i)* fails to repay, *(ii)* repudiates or imposes a moratorium, or *(iii)* restructures any of its borrowed money. This includes in particular any Treasury Bill, Bond or Note, whether nominal or indexed. In our empirical analysis, we identify default with the conditions for which CDS protection are triggered.

In the case of a credit event, the LGD is determined through an auction addressed to CDS dealer banks. Participating banks typically submit a bid and ask quote on a $100 face-value bond of the reference entity, and the cross-section of bid-asks is used to determine the final price of the bond, typically below par (see Augustin et al. (2014)).[12]

[12]The final price of the bond resulting from this auction is published by CreditEx (`http://www.creditfixings.com/CreditEventAuctions/results.jsp`).

Settlement of the CDS contract can be completed either through cash or physical delivery. In the former case, the protection seller delivers a payment equal to the LGD as determined by the auction, multiplied by the notional of the CDS. In the latter case, the protection seller pays the entire notional to the buyer in exchange for an equivalent principal amount of reference bonds. If these bonds have the exact same characteristics as those auctioned, the two deliveries would be equivalent. However, the protection buyer can choose to exchange *any of her reference bonds* with maturity below 30 years and above the maturity of the CDS contract. This essentially embeds a *cheapest-to-deliver* (CtD) option to the buyer's position, who will likely deliver the lowest dollar price reference obligation available.[13]

U.S. CDS contracts fall under the "Big Bang Protocol" established by ISDA in 2009. In the aftermath of the financial crisis, as the primary industry body overseeing swaps and derivative trading, ISDA pushed swap market participants to adopt the new protocol in an effort to standardize over-the-counter contract parameters.[14] A number of the implemented changes are worth highlighting. First, coupon payments on each contract are fixed at either 100 (investment grade) or 500 basis points (non-investment grade). As a result, there is typically a payment to be made at the initiation of the contract to ensure that the present values of expected cash flows are equal between the buyer's and seller's legs. A second important change stemming from the protocol is the hardwiring of the auction process following credit events such that all protection buyers obtain fair cash payments from protection sellers. Third, the protocol further stipulates the creation of Determinations Committees for determining whether a credit or succession event has occurred in order to reduce disputes between counterparties in case there is ambiguity.

Market participants in the sovereign CDS market include security dealers, banks and other financial institutions, and hedge funds (see e.g. Augustin (2018*a*)). There is evidence that sovereign CDS contracts are used in both a hedging and speculative context. For contracts specifically written on the U.S. sovereign, focusing on the most liquid contracts with five years to maturity, price data from Markit shows there is very little pricing movement before the financial crisis of 2008. The premium spiked in 2009, at the height of the crisis, to about 100 basis points and has remained elevated afterward between 20 to 40 basis points.

Chernov, Schmid and Schneider (2019) provide a detailed discussion on the determinants of

[13]In the context of the Greek crisis, CDS contracts and the associated auction mechanism played a minor role in the resturcturing process. As highlighted by Zettelmeyer, Trebesch and Gulati (2019), the credit event was triggered only after the preemptive debt restucturing. Therefore, the CDS auction took place after the bond exchange, and the resulting auction price fell in place with the new bond price in the secondary market. To be certain, CDS coverage of Greek sovereign debt was very low, at less than 2%. One would not expect the outcome of the bond auction to dictate terms of the restructuring.

[14]BIS Quarterly Review, December 2010. "The Big Bang in the CDS Market"

U.S. sovereign CDS spread beyond credit risk. For instance, the majority of U.S. CDS contracts are denominated in euros, and there is a small foreign exchange premium embedded in the spread. U.S. dollar denominated contracts did not start trading until August 2010 and volumes are thin relative to euro contracts. Additionally, there is uncertainty in the cheapest-to-deliver option due to the bond auction protocol conditional on default occurring. Lastly, the U.S. CDS spread should contain a liquidity premium component due to the relative scarcity of the instrument compared to other sovereign CDS contracts. A combination of these factors contribute to the U.S. sovereign CDS premium.

In the context of our project, we use the U.S. sovereign CDS premium as a proxy for default risk to study the relative pricing of nominal Treasury bonds and inflation-protected bonds. We present several robustness tests in the Appendix to rule out the possibility these non-credit risk-related factors can simultaneously generate differential prices in U.S. sovereign bonds.

2.3 CDS-Implied LGD and Effective LGD

In practice, there can be a significant difference in the LGD faced by uncovered and covered bond position holders. This discrepancy is due to the auction process determining the LGD used for CDS purposes. Let us assume that upon trigger of a U.S. sovereign credit event, the auction determines that the reference bond is worth 75 cents per dollar, yielding an auction-based LGD of 25 cents. In the case of physical delivery, an investor holding a covered position can sell her bond at par to the protection seller and receives one dollar.

An interesting case arises for cash delivery, where the protection seller delivers 25 cents to the protection buyer but the latter holds onto her bond. The government then determines an effective LGD which can be different than 25 cents. If the effective LGD is 20 cents, the protection buyer is left with one dollar and five cents. This effect is similar to the CtD option for physical delivery and can lead the CDS-implied LGD to be greater than the effective LGD for the bond holder.

While we are aware of the existence of these complications, we leave them aside in our empirical analysis for pragmatic reasons. Our rationale is that while it is straightforward to obtain the LGD resulting from a CDS auction for past credit events, obtaining the effective LGD from these events is a task with very little, if any data. In addition, the case of a U.S. credit event has been so rare that attempting to impute any figure would be pure conjecture. It should also be noted that we use CDS spreads merely as a proxy for the default risk of the U.S. sovereign, which allows us to abstract away from these specifics and assume that the CDS exactly embeds the effective LGD determined by the government.

3 Empirical Analysis

In this section we test our main hypothesis that exposure to default risk may influence the relative pricing of nominal and inflation-protected sovereign obligations. Specifically, we test whether the ILS-BEI spread is related to the CDS spread. We examine variation in these quantities over the full sample period and a subperiod that does not include the financial crisis of 2007-2009.

3.1 Data

The spreads between breakevens and inflation-linked swaps are constructed in two steps. We use the data described in Gurkaynak, Sack and Wright (2006) and Gurkaynak, Sack and Wright (2010) for nominal and inflation-protected smoothed zero-coupon bonds respectively. The BEI variable is the difference between the former and the latter. We collect inflation swap data from Bloomberg and subtract the BEI from the swap spread to obtain our mispricing variable, the ILS-BEI spread. EUR-denominated CDS spread data are obtained from Markit. Our focus is on the five-year maturity for CDS contracts as this is the most liquid CDS tenor. Our data are sampled daily from January 2008 to October 2015 (full sample).[15]

We depict the time series of U.S. sovereign credit default swap spreads and the ILS-BEI spread in Figure I, panel (b). As documented in Chernov, Schmid and Schneider (2019), CDS spreads soar to 100 basis points in the wake of the Lehman Brothers bankruptcy, timing that is similar to that of the large increase in ILS-BEI. Our conjecture is that this event, and the crisis that followed caused investors to reprice the probability of a U.S. sovereign default and the recovery on Treasury and TIPS in a default scenario. The spread is volatile in 2010-2013 before becoming quiescent from about 2014 onward. Notably, the spread spikes to more than 40 basis points in the days prior to the resolution of the the budget showdown of 2013, which threatened to lead to a U.S. sovereign default.

Summary statistics for these data are provided in Table I. Over the full sample period, both the ILS-BEI and U.S. CDS spread averaged over 30 basis points (36 and 33 basis points respectively). The ILS-BEI is approximately twice as volatile as the CDS spread, ranging from -1 to 210 basis points. In contrast to the CDS spread, the ILS-BEI declines both on average and in volatility in the post-crisis period, which we define as January 1, 2010 and beyond. Thus, even in the post-crisis

[15] Our results are qualitatively the same when using USD-denominated CDS contracts after they began trading in 2010. While data on EUR-denominated CDS are available prior to 2008, U.S. CDS exhibit virtually no variation and volume in the pre-sample period and the quotes are often unchanged for weeks at a time and average between one and two basis points.

period, the U.S. CDS spread averages 34 basis points, considerably greater than its pre-crisis levels. The unconditional correlation between the five-year ILS-BEI and CDS is about 0.3.

3.1.1 Regression controls

In addition to possible fears of default risk, numerous factors may play a role in the observed ILS-BEI spread — namely, heightened counterparty credit risk associated with inflation swap transactions, liquidity concerns, increases in perceived quantities and prices of risk, and a deterioration in arbitrage capital available to deploy in financial markets. Each of these potentially confounding factors would be expected to play an outsized role influencing the components of the ILS-BEI spread at the peak of the financial crisis. We examine the role of several variables in order to investigate alternative possibilities.

HPW Noise and *TIPS Noise* serve as our measures of arbitrage capital as proposed in Hu, Pan and Wang (2013)[16]. *LIBOR-OIS* measures counterparty credit risk. The off/on the run differential in nominal bonds (*OTR Difference*) is a proxy for liquidity in these markets while the *VIX* index is often viewed as a market measure of the prevailing price of risk in financial markets. A detailed description of each variable can be found in Appendix A.1.

3.2 Empirical Results

We employ panel regressions for our empirical analysis. We include the ILS-BEI spread across five tenors: 2, 3, 5, 7, and 10 years as the dependent variable. As is standard in the fixed income pricing literature, we assume that all interest rates at all maturities are second-order stationary despite a high persistence. Although stationarity tests usually fail to reject the presence of a unit root, they suffer from lack of power in small samples. In addition, it is difficult to justify that either the mean or variance of U.S. interest rates will follow an explosive path. Our baseline specification is thus given by:

$$(\mathrm{ILS}-\mathrm{BEI})_{n,t} = \alpha + \rho \cdot (\mathrm{ILS}-\mathrm{BEI})_{n,t-1} + \gamma \cdot \mathrm{CDS}_t + \boldsymbol{\beta}^\top \cdot \mathbf{X_t} + w_t + \varepsilon_{n,t}\,, \tag{2}$$

where $n = \{2, 3, 5, 7, 10\}$, $\mathbf{X_t}$ represents the set of relevant controls, and w_t is a week-time fixed effect. For ease of interpretation, we present the R^2 of this regression as the fraction of the ILS-BEI spread changes explained by our explanatory variables. This naturally brings the R^2 closer to zero, allowing us to see significant changes across specifications.

[16]The HPW Noise measure is sourced from Jun Pan's website and we thank Richard Crump for providing the TIPS Noise series.

Table II shows regression results for the full sample, spanning the beginning of 2008 to October 2015. We start the sample in 2008 due to the fact that U.S. sovereign CDS contracts were thinly traded prior to the 2008 financial crisis. All regressions contain observations at the daily frequency where data is available for consecutive trading days in all markets. Columns (1) - (6) depict results with individual covariates in the specification, and column (7) represents the full multivariate specification. In column (8) we add tenor fixed effects to the full specification. Finally, in each regression, the U.S. CDS spread is the main explanatory variable, but we include the lagged ILS-BEI spread to ensure the persistence of the dependent variable is not driving our results.

As shown in the top row of Table II, all coefficient loadings on the U.S. CDS spread are positive and highly significant across the columns. The point estimate of 0.196 in Column (1) is statistically significant at the 1% level and suggests that a 16 basis point increase in U.S. CDS spreads (one standard deviation) translates into an approximately 3.1 basis point increase in the ILS-BEI spread. This represents approximately 10% of the mean ILS-BEI suggesting the results are economically significant as well.

Table II also demonstrates that it is essential to include lagged ILS-BEI spread as an explanatory variable since coefficient loadings are positive and highly significant regardless which control variables are used. It should not be surprising that the ILS-BEI spread is highly persistent. Amongst the control variables, LIBOR-OIS spread in Column (4), and VIX in Column (6) are marginally significant (between 5% and 10% statistical significance). The TIPS Noise measure in Column (3) and the OTR Difference in Column (5) are negative and highly significant. A higher value of TIPS Noise indicates greater deviations in the TIPS yield curve. Although we would expect this to reflect poorer liquidity on the TIPS market and thus to increase the ILS-BEI spread, we note that all liquidity measures are highly correlated and it is hard to extract a clean interpretation for each single coefficient. When all variables are included, in Column (7) of Table II, the coefficient on CDS increases to 0.228 (3.6bps per CDS standard deviation), and HPW Noise becomes statistically significant. Given the high positive correlation with TIPS Noise this result is not surprising. The estimated coefficient loading on the CDS spread actually increases from Column (1) to Column (7) when controls are included. Finally, the addition of a tenor fixed effect does not affect the regression outcomes in Column (8) suggesting our results are not driven by a particular maturity on the yield curve. We present the fitting measure as one minus the ratio of unexplained variance over the variance of the first difference of ILS-BEI spreads. These spreads are highly persistent and standard R^2 measures are close to one when controlling for lagged spreads, thus barely informative. We use this measure throughout for all regressions in levels using the lagged ILS-BEI spread as a control.

We repeat our main empirical test in first differences rather than levels to further ensure the persistence of ILS-BEI is not driving our finding and report the results in Table III. Similar to Table II, the change in CDS spread is shown to be a positive and significant factor driving the change in ILS-BEI in the first row. Under the full specification in Column (8), with both week and tenor fixed effects, a 1% increase in the change of CDS spread results in a 16 bps greater increase in the change of ILS-BEI after controlling for changes in the noise measures, systemic risk, liquidity and volatility. In line with Table II, changes in TIPS noise and changes in OTR Differences are negative and highly significant under the full specification, suggesting the liquidity effect is present in first differences as well. Results presented in Table III should ease the concern that the relationship between CDS spread and ILS-BEI we document is purely spurious.

To check that the credit risk influence indeed stems from TIPS, we regress the components of ILS-BEI separately on U.S. CDS spread in the full sample. Table IV documents the regression results. The dependent variables in columns (1), (2), and (3) are, respectively, TIPS yields, nominal Treasury yields, and ILS swap premia. To be consistent with previous tests, we employ tenors of 2-, 3-, 5-, 7-, and 10-years on all the dependent variables. The explanatory variables include the 5-year CDS spread, lagged dependent variables, as well as standard controls used in Table II.

Both TIPS yields and nominal yields load positively and significantly on the CDS spread, in Columns (1) and (2), whereas the ILS spread shows no significant correlation with the CDS spread in Column (3). This suggests that the ILS-BEI spread comoves with the CDS spread because of the reaction of the real and nominal term structures to sovereign default risk. Moreover, the coefficient loading of TIPS yields on CDS is larger than the coefficient loading of nominal yields on CDS. This implies that the BEI narrows as CDS spread increase, and that the ILS-BEI spread increases.

Results shown in Table IV provide some comfort that the CDS spread is indeed producing differential impact on sovereign bond yields and not on the inflation swap. This is crucial evidence that sovereign default risk can impact the relative pricing of TIPS and nominal bonds.

3.3 Sub-sample Analysis

We examine the degree to which the 2008 financial crisis influences our conclusions by separating the sample into a crisis period, which we specify as January, 2008 through December, 2009, and a post-crisis period from January, 2010 onward. Results for the crisis period are presented in Table V. Our results carry through during the crisis sample. Depending on the specification, CDS coefficients range from 0.20 to 0.27. Although their statistical significance is weaker (5-10% level) during the crisis period relative to the full sample, the point estimates on the U.S. CDS spread are greater, which implies a more pronounced effect between sovereign default risk and first differences in the ILS-BEI spread.

For the post-crisis period, the results depicted in Table VI are essentially unchanged as well. Again, depending on the specification, CDS coefficients range from 0.16 to 0.17 and are significant at the 1% level, indicating more precise estimates than in the crisis sample. The point estimates across all columns are roughly five times greater than their standard errors.

Our interpretation of these results is that determinants of the U.S. CDS spread comove strongly with daily changes in ILS-BEI. While CDS spreads may be driven by a number of different factors, including actual default risk and liquidity effects, we view the evidence here as sufficiently suggestive to indicate ILS-BEI is influenced by credit risk. The relationship appears to persist during periods of high volatility and strained financial conditions (crisis) as well under conditions associated with normally functioning markets.

3.4 Default Risk and Liquidity

Pflueger and Viceira (2016) suggest that much of the spread between nominal and inflation-protected bond yields arises as a premium for liquidity. In their analysis, they find that the portion of breakeven inflation that is related to liquidity rather than inflation expectations accounts on average for 69bps of the spread between nominal and inflation-protected securities. While we endeavor to control for liquidity in our earlier analysis, in this section we explicitly examine the contribution of CDS to the liquidity premium that they document.

The authors measure the liquidity premium by breaking the differential in the yield on nominal and inflation-protected securities on a set of liquidity variables and measures of inflation expectations:

$$\text{BEI}_t^{(n)} = a_1 + \mathbf{a}_2^\top \mathbf{X}_t + \mathbf{a}_3^\top \boldsymbol{\pi}_t^e + \varepsilon_{n,t}, \tag{3}$$

where $\mathbf{X}_t$ is a vector of liquidity-related variables and $\boldsymbol{\pi}_t^e$ is a vector of measures of inflation expectation. The liquidity premium is measured as $\hat{L}_t = -\hat{\mathbf{a}}_2^\top \mathbf{X}_t$. We follow their approach, using the breakeven inflation between 10-year nominal and inflation-protected securities as our dependent variable. We describe the independent variables in Appendix A.2.1. One key point is that the liquidity variables $\mathbf{X}_t$ include the ILS-BEI spread as a proxy.

Results of the analysis are presented in Table VII. In the first column of Panel A, we present an analysis complementary to that of Pflueger and Viceira (2016). Consistent with their analysis, and with intuition, inflation expectation variables are positively related to breakeven inflation. The coefficient on the ILS-BEI is negative; the authors interpret the result as suggesting that the pronounced decrease in breakeven inflation during the financial crisis reflected security market disruption and constraints on levered investors. The authors find that one cannot reject the hypothesis that the coefficient is equal to negative one. However, in our results, the point estimate of the coefficient is more than two standard errors from one. This result suggests that, consistent with our results above, the ILS-BEI may reflect more than just constraints on levered market participants.

In the second column, we add the CDS spread to the regression. Three observations emerge. First, the CDS spread is negatively and significantly related to breakeven inflation. To the extent that default risk may have differential impact on nominal and inflation-protected Treasury securities, the negative coefficient suggests that yield spreads on the two securities tighten when default risk increases. This may reflect a flight to the relative safety of nominal Treasuries or a drop in the prices of inflation-protected securities. Second, the coefficients on the remaining variables, with the exception of ILS-BEI, are materially unaffected. Third, after controlling for CDS, one can no longer reject the hypothesis that the coefficient on the ILS-BEI is equal to negative one, consistent with the results in Pflueger and Viceira (2016). Thus, the results indicate that both the BEI and the ILS-BEI reflect co-movement with CDS spreads due to credit risk.

Our final analysis of the liquidity premium directly regresses the estimated liquidity premium on the CDS spread. Results are presented in Panel B. As shown in the Table, CDS spreads on Treasury securities are positively and statistically significantly related to the liquidity premium, explaining approximately 9% of its variation. This result suggests that part of the liquidity premium documented in Pflueger and Viceira (2016) may in fact reflect compensation for credit risk. However, the majority of the variation in the premium is unrelated to variation in CDS spreads, indicating that both liquidity and credit risk jointly play a role in understanding the x pricing of TIPS and nominal Treasury securities.

3.5 United Kingdom Evidence on Default Risk and Bond Pricing

In this section, we validate our hypothesis in the international setting. The United Kingdom was one of the first major developed markets to introduced inflation-linked bonds in 1981. These "linkers" are much like TIPS in the U.S. in that their principal and coupons are contractually linked to a domestic price index acting as real bonds in contrast to U.K. gilts. Thus, much as we see different responses in U.S. nominal and real yields to default risk, we expect similar patterns in the relation between hedged gilts, linkers, and CDS spreads in the U.K. For the U.K. analysis, we focus only on the post crisis period because the subsample analysis in Section 3.3 shows that the relationship between ILS-BEI and CDS spread is statistically stronger after 2009. This means the impact of sovereign credit risk on bond yields is not a crisis period phenomenon.

We obtain U.K. yield data from the Bank of England's website and CDS spreads from Markit. We perform the same panel regressions as those in Table VI by pooling yields across tenors at the daily frequency for the post-financial crisis sample period from January 2010 to October 2015. Where data are available, we utilize U.K. control variables similar to those employed in our U.S. analysis. Specifically, we control for the off/on the run differential in nominal bonds, GBP LIBOR-OIS spread, and VSTOXX index (Euro Stoxx 50 volatility index), but do not have access to data for a measure comparable to the HPW noise variable.

Regression results for the U.K. are reported in Table VIII. We control for lagged ILS-BEI just as we have done for the U.S. In Column (1), we see U.K. CDS spread is positive and significantly related to contemporaneous ILS-BEI. A one percent increase in the CDS spread translates to a 10 bps widening of the ILS-BEI. The magnitude of the coefficient on U.K. CDS declines to about 7 bps in Column (5) with the VSTOXX index as a control. Results in the full specification in Column (6) remain little changed. The U.K. results are consistent with those in Table VI for the U.S.: higher default risk as proxied by larger CDS spread leads to a widening of the ILS-BEI in the data.

Table IX presents the U.K. regression results in first differences rather than in levels. These regressions mirror those shown in Table III for the U.S. The dependent variable is the one-period change in the U.K. ILS-BEI, and the main explanatory variable is the one-period change in the U.K. CDS spread. All control variables are also employed in first differences. In Column (1), a larger innovation in the CDS spread implies a greater increase in the ILS-BEI. The estimated coefficient is statistically significant at the 1% level. The statistical significance in the first row remains in subsequent columns as control variables are added in first differences. In Column (6) of Table IX, with all the controls as well as both week and tenor fixed effects, the change in U.K. ILS-BEI still loads positive and significantly on the change in U.K. CDS spread. Overall, our U.K. findings corroborate with those in the U.S.: higher sovereign default risk leads to a narrowing of the risky break even inflation as real bonds decline more in price than nominal bonds.

4 Modeling Nominal and Inflation-Protected Debt with Default Risk

In this section, we discuss the pricing of nominal and inflation-protected sovereign bonds, assuming that there is a possibility of a credit event interrupting the promised payments of the securities. Of particular interest is the spread between inflation-linked swaps and the breakeven inflation rate. We develop a new affine pricing model to complement panel regression results in Subsection 3.2. With the aid of the model, we decompose the ILS-BEI spread into its credit and liquidity components to examine the dynamic contribution of the credit factors during the after the crisis.

4.1 Riskless yields dynamics

Let us consider a vector of N_x unobservable factors, denoted by x_t. We assume these factors have standard Gaussian VAR dynamics such that:

$$x_t = \mu + \Phi x_{t-1} + \sqrt{\Sigma}\varepsilon_t \quad \text{where} \quad \varepsilon_t \overset{i.i.d}{\sim} \mathcal{N}\left(0,\, I_{N_x}\right)\,. \tag{4}$$

In this economy, there exists a riskless nominal asset trading at date t at price e^{-r_t} and delivering one unit of cash at $t+1$. We assume that the riskless nominal yield is given by a linear combination of x_t:

$$r_t = \kappa_0^{(r)} + \kappa_x^{(r)\prime} x_t\,, \tag{5}$$

where $\kappa_0^{(r)}$ is a scalar and $\kappa_x^{(r)}$ is a vector of size N_x. The term structure of riskless nominal yields will therefore be entirely driven by x_t. These yields will in turn constitute our discount rates for the remaining nominal securities (see for instance Duffie and Kan (1996)).

4.2 Default and liquidity dynamics

Our modeling framework follows that of Monfort et al. (2020) in modeling risky debt in discrete time. In their framework, sovereign credit events of any kind are represented by jumps of a non-negative credit-event variable denoted by $\delta_t^{(c)}$. Our modeling of liquidity events on the TIPS market mimics the form employed for credit events (see e.g. Ericsson and Renault (2006), Monfort and Renne (2013) or Dubecq et al. (2016)). We assume that liquidity events on the TIPS market are represented by jumps of a liquidity-event variable denoted by $\delta_t^{(\ell)}$. More formally, the times of credit and liquidity events, τ_c and τ_ℓ, can be defined as:

$$\tau_c = \min\left\{\, t \,|\, \delta_t^{(c)} > 0 \right\} \quad \text{and} \quad \tau_\ell = \min\left\{\, t \,|\, \delta_t^{(\ell)} > 0 \right\}. \tag{6}$$

Note that, since we do not observe any default event during our sample, the time series of $\delta_t^{(c)}$ will be uniformly zero throughout the sample. We assume the same for the liquidity event variable $\delta_t^{(\ell)}$ for simplicity.[17] The probabilities of credit and liquidity events are driven by the default and liquidity intensities $\lambda_t^{(c)}$ and $\lambda_t^{(\ell)}$, respectively. We assume that the credit- and liquidity-event variables are Gamma-zero distributed given their respective intensities. That is, there exist Poisson distributed random variables $\mathsf{P}_t^{(c)}$ and $\mathsf{P}_t^{(\ell)}$ such that:[18]

$$\begin{cases} \mathsf{P}_t^{(c)} \,\big|\, \lambda_t^{(c)} \sim \mathcal{P}\left(\lambda_t^{(c)}\right) & \text{and} \quad \delta_t^{(c)} \,\big|\, \mathsf{P}_t^{(c)} \sim \Gamma_{\mathsf{P}_t^{(c)}}\left(\mathsf{c}_\delta^{(c)}\right), \\ \mathsf{P}_t^{(\ell)} \,\big|\, \lambda_t^{(\ell)} \sim \mathcal{P}\left(\lambda_t^{(\ell)}\right) & \text{and} \quad \delta_t^{(\ell)} \,\big|\, \mathsf{P}_t^{(\ell)} \sim \Gamma_{\mathsf{P}_t^{(\ell)}}\left(\mathsf{c}_\delta^{(\ell)}\right), \end{cases} \tag{7}$$

where $\mathsf{c}_\delta^{(c)}$ and $\mathsf{c}_\delta^{(\ell)}$ are positive scaling parameters, and $\mathsf{P}_t^{(c)}$ and $\mathsf{P}_t^{(\ell)}$ are the degree of freedom parameters, at date t, of the associated gamma distribution. With the assumption of Equation (7), it is easy to see that the survival probabilities are respectively given by $e^{-\lambda_t^{(c)}}$ and $e^{-\lambda_t^{(\ell)}}$. Monfort et al. (2020) show that Gamma-zero processes are efficient in representing credit events since they can stay at the value of zero for extended periods of time (no default or liquidity states) and jump to any positive value upon events. In the following, we use the notation $\delta_t = \left(\delta_t^{(c)},\, \delta_t^{(\ell)}\right)'$ and $\Gamma_0\left(\lambda_t,\, \mathsf{c}_\delta\right)$ for the Gamma-zero distribution.

Staying true to the spirit of affine models, we assume that credit and liquidity intensities are linearly related to N_c and N_ℓ vectors of state variables, denoted by $y_t^{(c)}$ and $y_t^{(\ell)}$ respectively. :

$$\lambda_t^{(c)} = \beta_\lambda^{(c)\prime} y_t^{(c)} \qquad \text{and} \qquad \lambda_t^{(\ell)} = \beta_\lambda^{(\ell)\prime} y_t^{(\ell)}, \tag{8}$$

where $\beta_\lambda^{(c)}$ and $\beta_\lambda^{(\ell)}$ are vectors of non-negative entries with size N_c and N_ℓ, respectively. To ensure that these intensities are positive, we assume that the vector $y_t = \left(y_t^{(c)\prime},\, y_t^{(\ell)\prime}\right)'$ of size $N_y = N_c + N_\ell$ is a vector autoregressive gamma process:

$$y_t \,\big|\, y_{t-1} \sim \Gamma_\nu\left(\beta_y\, y_{t-1}; \mathsf{c}_y\right), \tag{9}$$

where ν is the vector of N_y positive degree of freedom parameters, c_y is a vector of N_y positive scaling parameters, and β_y is a $N_y \times N_y$ matrix with non-negative entries representing the potential Granger causality between the credit and liquidity components of y_t.

The risk factors x_t, y_t and δ_t constitute the entirety of state variables in this economy. Monfort et al. (2020) show that the stacked vector $w_t = (x_t', y_t', \delta_t')'$ of size $N = N_x + N_y + 2$ is affine, such that its conditional moment-generating function is exponential-affine and that it follows a

[17] Note that, this makes it harder for the model to fit the TIPS yields, if anything. Indeed, by imposing that $\delta_t^{(\ell)} = 0$ during our sample, we effectively suppress one factor for the fit.

[18] See Monfort et al. (2017) for details on the gamma-zero process.

semi-strong heteroskedastic VAR:

$$w_t = \Psi_0 + \Psi w_{t-1} + \sqrt{\Omega_{t-1}}\,\xi_t, \tag{10}$$

where ξ_t is a martingale difference with unit variance, and Ψ_0, Ψ and Ω_{t-1} are detailed in Appendix A.3.

4.3 Inflation dynamics

In our economy, agents have access to inflation-indexed assets that compensate them for inflation fluctuations. We assume that inflation π_t is defined by the monthly log variation of the unadjusted consumer price index (CPI-U) between $t-1$ and t. Its dynamics are given by:

$$\pi_t = \kappa_0^{(\pi)} + \kappa_x^{(\pi)\prime} x_t + \kappa_y^{(\pi)\prime} y_t^{(c)} + \kappa_\delta^{(\pi)} \delta_t^{(c)}\,, \tag{11}$$

where $\kappa_0^{(\pi)}$ is a scalar, $\kappa_x^{(\pi)}$ and $\kappa_y^{(\pi)}$ are vectors of size N_x, and N_c, respectively, and $\kappa_\delta^{(\pi)}$ is a scalar. Our specification for inflation contains three key components. First, inflation has a Gaussian component $\kappa_x^{(\pi)\prime} x_t$, mimicking the standard affine term structure models for TIPS (see e.g. Abrahams, Adrian, Crump and Moench (2016)). Second, our inflation process depends on the factors driving the default intensity through $\kappa_y^{(\pi)\prime} y_t^{(c)}$, allowing to capture its possible correlation with distance to default. In particular, the possibility of inflating away the debt would translate into significant components of $\kappa_y^{(\pi)}$. Last, the specification of Equation (11) allows for a hyperinflation or deflation jump upon default when $\kappa_\delta^{(\pi)}$ is positive or negative, respectively, reproducing the potential stigma associated with sovereign default.

4.4 The stochastic discount factor

We assume that no-arbitrage holds such that there exists a (nominal) stochastic discount factor M_{t+1} associated with the representative investor in this economy. We assume that its specification is given by:

$$M_{t+1} = \exp\left(-r_t + \theta_{x,t}'\,\varepsilon_{t+1} + \theta_y' y_{t+1} - \frac{1}{2}\theta_{x,t}'\,\Sigma\,\theta_{x,t} - \left[\frac{\text{diag}(\theta_y)\mathsf{c}_y}{\mathbf{1} - \text{diag}(\theta_y)\mathsf{c}_y}\right]'\beta_y\,y_t + \nu' \log\left[\mathbf{1} - \text{diag}(\theta_y)\mathsf{c}_y\right]\right), \tag{12}$$

where $\theta_{x,t} = \theta_{0,x} + \Theta_x\,x_t$, each component of θ_y is in $(0, 1/\mathsf{c}_y)$, and the last three terms appear such that the no-arbitrage relationship $\mathbb{E}_t(M_{t+1}) = e^{-r_t}$ is verified. We show in Appendix A.4 that this stochastic discount factor is structure-preserving, such that the classes of distribution are the same, with shifted parameters. More specifically, we still have that $\delta_t \overset{\mathbb{Q}}{\sim} \Gamma_0(\lambda_t, \mathsf{c}_\delta)$, the riskless factors x_t follow a Gaussian VAR given by:

$$x_t = \mu^{\mathbb{Q}} + \Phi^{\mathbb{Q}} x_{t-1} + \sqrt{\Sigma}\, \varepsilon_t^{\mathbb{Q}} \quad \text{where} \quad \varepsilon_t^{\mathbb{Q}} \overset{i.i.d.}{\sim} \mathcal{N}(0, I_{N_x}) \,, \tag{13}$$

and the credit and liquidity factors have modified vector autoregressive gamma dynamics, such that:

$$y_t \,\big|\, y_{t-1} \overset{\mathbb{Q}}{\sim} \Gamma_\nu \left(\beta_y^{\mathbb{Q}}\, y_{t-1}; \mathsf{c}_y^{\mathbb{Q}}\right) \,. \tag{14}$$

The transition between risk-neutral and physical parameters is closed-form and given by:

$$\mu^{\mathbb{Q}} = \mu + \Sigma\, \theta_{0,x}, \quad \Phi^{\mathbb{Q}} = \Phi + \Sigma\, \Theta_x, \quad \beta_y^{\mathbb{Q}} = \beta_y \operatorname{diag}\left(\frac{\mathbf{1}}{\mathbf{1} - \operatorname{diag}(\theta_y)\, \mathsf{c}_y}\right), \quad \mathsf{c}_y^{\mathbb{Q}} = \frac{\mathsf{c}_y}{\mathbf{1} - \operatorname{diag}(\theta_y)\, \mathsf{c}_y} \,. \tag{15}$$

For all asset prices and yields below, we will have to compute the risk-neutral expectation of exponential-affine combinations of w_t, that is of x_t, y_t and δ_t. For convenience, using the properties of affine processes, we introduce the following notation:

$$\mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(u' \sum_{j=1}^{n-1} w_{t+j} + v'\, w_{t+n}\right)\right] =: \exp\left[\mathcal{A}_n^{\mathbb{Q}}(u,v) + \mathcal{B}_n^{\mathbb{Q}}(u,v)'\, w_t\right] \,, \tag{16}$$

where $u \in \mathbb{R}^N$ and $v \in \mathbb{R}^N$ are two arguments that will vary depending on the asset to price, and $\mathcal{A}_n^{\mathbb{Q}}(u,v)$ and $\mathcal{B}_n^{\mathbb{Q}}(u,v)$ are parametric functions that depend on all risk-neutral parameters. These function are detailed in Appendix A.4.

4.5 The term structure of riskless yields

In our economy, the representative investor has access to riskless nominal and real zero-coupon bonds providing one unit of cash and one consumption unit at maturity, respectively. Nominal zero-coupon bonds with residual maturity n trade at time t at price $D_t^{(n)}$, such that $D_t^{(1)} = e^{-r_t}$. Real bonds issued at date t trade at price $D_t^{*(n)}$ and provide the compounded inflation $e^{\sum_{j=1}^{n} \pi_{t+j}}$ at date $t+n$. In our framework, both nominal and real riskless bond prices are closed-form functions of the state vector. Denoting by $\kappa^{(r)}$ and $\kappa^{(\pi)}$ the size-N vectors such that $r_t = \kappa_0^{(r)} + \kappa^{(r)\prime} w_t$ and

$\pi_t = \kappa_0^{(\pi)} + \kappa^{(\pi)\prime} w_t$, we show in Appendix A.5 that:

$$\begin{aligned} D_t^{(n)} &= \exp\left\{-n\kappa_0^{(r)} + \mathcal{A}_n^{\mathbb{Q}}\left(-\kappa^{(r)}, \mathbf{0}\right) + \left[\mathcal{B}_n^{\mathbb{Q}}\left(-\kappa^{(r)}, \mathbf{0}\right) - \kappa^{(r)}\right]' w_t\right\} \\ D_t^{*(n)} &= \exp\left\{-n\left(\kappa_0^{(r)} - \kappa_0^{(\pi)}\right) + \mathcal{A}_n^{\mathbb{Q}}\left(\kappa^{(\pi)} - \kappa^{(r)}, \kappa^{(\pi)}\right) + \left[\mathcal{B}_n^{\mathbb{Q}}\left(\kappa^{(\pi)} - \kappa^{(r)}, \kappa^{(\pi)}\right) - \kappa^{(r)}\right]' w_t\right\}. \end{aligned} \tag{17}$$

Note that due to our distributional assumptions and the fact that r_t is a function of x_t only, riskless nominal bond prices are functions of x_t (identifying x_t as "riskless" factors), while real riskless bond prices can be a function of the credit risk factors $y_t^{(c)}$ through the inflation specification. Equation (17) produces two key features of our model for riskless yields. First, both nominal and real yields are affine functions of w_t, thus the model is an affine term structure model (ATSM). Second, zero-coupon inflation swap rates are also closed-form affine functions of w_t, given by:

$$\text{ILS}_t^{(n)} = \kappa_0^{(\pi)} + \frac{\mathcal{A}_n^{\mathbb{Q}}\left(\kappa^{(\pi)} - \kappa^{(r)}, \kappa^{(\pi)}\right) - \mathcal{A}_n^{\mathbb{Q}}\left(-\kappa^{(r)}, \mathbf{0}\right)}{n} + \frac{\mathcal{B}_n^{\mathbb{Q}}\left(\kappa^{(\pi)} - \kappa^{(r)}, \kappa^{(\pi)}\right)' - \mathcal{B}_n^{\mathbb{Q}}\left(-\kappa^{(r)}, \mathbf{0}\right)'}{n} w_t \,. \tag{18}$$

4.6 Recovery assumptions

While the representative investor has access to the riskless assets presented above, she can also invest in nominal and real Treasuries (TIPS) issued by the sovereign government. We assume both types of Treasuries are subject to credit risk, while only TIPS are subject to liquidity risk for simplicity. Before turning to their respective pricing, we detail the recovery earned by the owner of each bond for both types of events.

Upon the trigger of a credit event ($\delta_{\tau_c}^{(c)} > 0$), we assume that both nominal bonds and TIPS are terminated, so that there is no selective default. We assume that the most likely situation in case of a credit event in the U.S. is that debt will be restructured, in a similar fashion to what happened in Greece in 2012. Thus, in this case, the U.S. government will offer investors to exchange the face value of outstanding bonds, nominal or inflation-indexed, against the same face value of a newly issued reference bond. While we cannot know *ex-ante* what this (or these) reference bond(s) are, we choose a nominal bond of maturity n_r, where n_r is longer than most traded sovereign bonds. In practice, we will set $n_r = 20$ years. A bond with long maturity will produce a low enough price such that most investors will suffer a significant loss given default by being to exchange short duration bonds against ones with a longer duration.

The reference bond has value $\mathcal{P}_t^{(n_r)}$ at date t, and features recovery of market value in case of a credit event (see Duffie and Singleton (1999)), with recovery rate $e^{-\delta_t^{(c)}}$. This means that at time of default, the credit-event variable $\delta_t^{(c)}$ jumps up by a magnitude representing the loss given default of the reference bond. Monfort et al. (2020) show that the price can be expressed as:

$$\mathcal{P}_t^{(n_r)} = \exp\left\{-n_r\kappa_0^{(r)} + \mathcal{A}_{n_r}^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathbf{e}_c, -\mathbf{e}_c\right) + \left[\mathcal{B}_{n_r}^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathbf{e}_c, -\mathbf{e}_c\right) - \kappa^{(r)} - \mathbf{e}_c\right]' w_t\right\}, \quad (19)$$

where $\mathbf{e}_c$ is the vector such that $\delta_t^{(c)} = \mathbf{e}_c' w_t$. We will write $B_t^{(n_r)} =: \exp\left(\mathsf{A}_{n_r} + \mathsf{B}_{n_r}' w_t\right)$ in the following sections.

For other nominal Treasuries of maturity n, we assume that the payment in case of a credit event is exactly $\mathcal{P}_{\tau_c}^{(n_r)}$, such that face values get exchanged one-for-one. For TIPS, we assume that the sovereign government can disindex the face value from realized past inflation, either partially or fully. We denote by $\rho^* \in (0,1)$ the parameter controlling the degree of indexation of TIPS, such that the recovery payment of TIPS are given by $\exp\left(\rho^* \sum_{j=t+1}^{\tau_c} \pi_j\right) \mathcal{P}_{\tau_c}^{(n_r)}$, for a bond issued at date t. If $\rho^* = 1$, there is full indexation and the sovereign government fully honors inflation compensation. In turn, if $\rho^* = 0$, there is full disindexation and the government completely forgives inflation indexation, in which case the recovery payment for nominals and TIPS becomes identical.[19]

Last, in the case of a TIPS liquidity event ($\delta_{\tau_\ell}^{(\ell)} > 0$), we assume that TIPS are terminated and provide a recovery payment of $e^{-\delta_{\tau_\ell}^{(\ell)}}$ per unit of inflated face value, i.e. $e^{-\delta_{\tau_\ell}^{(\ell)} + \pi_{t+1} + \ldots + \pi_{\tau_\ell}}$. Thus, the magnitude of the $\delta_{\tau_\ell}^{(\ell)}$ jump represents the severity of the rebate of a TIPS sold on the secondary market.[20]

[19] To preserve the absence of arbitrage opportunities, we assume that the reference bond is not directly tradable by investors and is not part of the regular trading curve of sovereign nominals. One can thus view the formulation of Equation (19) as a pure reduced-form assumption where the recovery rate of any bond is given by $\mathcal{P}_t^{(n_r)}$, a quantity close to the price of a nominal bond of maturity n_r.

[20] While defining the recovery payment as a fraction of the price that would have prevailed in the absence of liquidity event (RMV) would be more realistic, it is not possible to obtain closed-form pricing formulas with that assumption in our framework. As noted by Duffie and Singleton (1999), differences between RMV and RFV assumptions tend to be small empirically, and this is unlikely to have a significant impact on our results.

4.7 The term structure of sovereign Treasuries

Having defined recovery payments in case of default and liquidity events, we turn to pricing the term structures of sovereign bonds. The price of nominal bonds is given by:

$$B_t^{(n)} = \sum_{i=1}^{n} \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(-\sum_{j=0}^{i-1} r_{t+j} \right) \mathcal{P}_{t+i}^{(n_r)} \times \left(\mathbb{1}\left\{ \sum_{j=0}^{i-1} \delta_{t+j}^{(c)} = 0 \right\} - \mathbb{1}\left\{ \sum_{j=0}^{i} \delta_{t+j}^{(c)} = 0 \right\} \right) \right] + \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(-\sum_{j=0}^{n-1} r_{t+j} \right) \mathbb{1}\left\{ \sum_{j=0}^{n} \delta_{t+j}^{(c)} = 0 \right\} \right]. \quad (20)$$

Equation (20) simply states that the price of the nominal bond is the sum of discounted recovery payments if default happens between $t+i-1$ and $t+i$ (first row), and the discounted principal if no default occurs during the lifespan of the bond (second row). An inflation-indexed bond is priced similarly, adding the possibility of liquidity risk. The price of the inflation-indexed bond is then given by:[21]

$$\begin{aligned} B_t^{*(n)} = & \sum_{i=1}^{n} \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(-\sum_{j=0}^{i-1} r_{t+j} - \rho^* \pi_{t+j+1} \right) \mathcal{P}_{t+i}^{(n_r)} \times \left(\mathbb{1}\left\{ \sum_{j=0}^{i-1} \delta_{t+j}^{(c)} = 0 \right\} - \mathbb{1}\left\{ \sum_{j=0}^{i} \delta_{t+j}^{(c)} = 0 \right\} \right) \right] \\ & + \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(-\sum_{j=0}^{i-1} r_{t+j} - \pi_{t+j+1} \right) e^{-\delta_{t+i}^{(\ell)}} \times \mathbb{1}\left\{ \sum_{j=0}^{i} \delta_{t+j}^{(c)} = 0 \right\} \left(\mathbb{1}\left\{ \sum_{j=0}^{i-1} \delta_{t+j}^{(\ell)} = 0 \right\} - \mathbb{1}\left\{ \sum_{j=0}^{i} \delta_{t+j}^{(\ell)} = 0 \right\} \right) \right] \\ & + \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(-\sum_{j=0}^{n-1} r_{t+j} - \pi_{t+j+1} \right) \mathbb{1}\left\{ \sum_{j=0}^{n} \delta_{t+j}^{(c)} = 0 \right\} \mathbb{1}\left\{ \sum_{j=0}^{n} \delta_{t+j}^{(\ell)} = 0 \right\} \right]. \end{aligned} \quad (21)$$

Equation (21) decomposes the price of a TIPS into the discounted recovery payment in case of default (first row), the discounted liquidity recovery in case of a liquidity event and no default event (second row), and the discounted inflated face value at maturity if no default and liquidity events happen (last row).

[21] We leave aside the embedded option and the inflation lag for simplicity in these pricing formulas. First, note that the embedded inflation option would, if anything, raise the price of the TIPS, decrease its yield, thus play against a large ILS-BEI spread. Hence, by neglecting the deflation option, we underestimate the role of the other factors, if anything. In addition, notice that the value of this deflation floor would be the biggest during the financial crisis, where the ILS-BEI spread is the biggest. Our simplification is thus conservative. Second, while the inflation lag can matter for short-enough maturities (below 2y), it is unlikely to have a large impact for longer maturities since the 3-months lag represents a smaller proportion of the total maturity of the bond.

We show in Appendix A.6 that our model provides closed-form pricing formulas for Equations (20-21), such that the price of a zero coupon nominal Treasury of residual maturity n is given by:

$$B_t^{(n)} = \lim_{\mathsf{u}\to+\infty} e^{\mathsf{A}_{n_r} - \kappa^{(r)\prime} w_t} \sum_{i=1}^{n} e^{-i\kappa_0^{(r)}} \left[\exp\left\{ \mathcal{A}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, \mathsf{B}_{n_r}\right) + \mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, \mathsf{B}_{n_r}\right)' w_t \right\} \right.$$
$$\left. - \exp\left\{ \mathcal{A}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, \mathsf{B}_{n_r} - \mathsf{u}\mathbf{e}_c\right) + \mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, \mathsf{B}_{n_r} - \mathsf{u}\mathbf{e}_c\right)' w_t \right\} \right]$$
$$+ \exp\left\{ -n\kappa_0^{(r)} + \mathcal{A}_n^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, -\mathsf{u}\mathbf{e}_c\right) + \left[\mathcal{B}_n^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, -\mathsf{u}\mathbf{e}_c\right) - \kappa^{(r)}\right]' w_t \right\}, \quad (22)$$

where the limit can be obtained numerically by putting a large scalar argument u, and for TIPS we have:

$$B_t^{*(n)} = \lim_{\mathsf{u}\to+\infty} \quad e^{\mathsf{A}_{n_r} - \kappa^{(r)\prime} w_t} \sum_{i=1}^{n} e^{i\left(\rho^*\kappa_0^{(\pi)} - \kappa_0^{(r)}\right)} \left[\exp\left\{ \mathcal{A}_i^{\mathbb{Q}}\left(\rho^*\kappa^{(\pi)} - \kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, \rho^*\kappa^{(\pi)} + \mathsf{B}_{n_r}\right) \right.\right.$$
$$\left. + \quad \mathcal{B}_i^{\mathbb{Q}}\left(\rho^*\kappa^{(\pi)} - \kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, \rho^*\kappa^{(\pi)} + \mathsf{B}_{n_r}\right)' w_t \right\}$$
$$\left. - \exp\left\{ \mathcal{A}_i^{\mathbb{Q}}\left(\rho^*\kappa^{(\pi)} - \kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, \rho^*\kappa^{(\pi)} + \mathsf{B}_{n_r} - \mathsf{u}\mathbf{e}_c\right) + \mathcal{B}_i^{\mathbb{Q}}\left(\rho^*\kappa^{(\pi)} - \kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, \rho^*\kappa^{(\pi)} + \mathsf{B}_{n_r} - \mathsf{u}\mathbf{e}_c\right)' w_t \right\} \right]$$
$$+ e^{-\kappa^{(r)\prime} w_t} \sum_{i=1}^{n} e^{i\left(\kappa_0^{(\pi)} - \kappa_0^{(r)}\right)} \left[\exp\left\{ \mathcal{A}_i^{\mathbb{Q}}\left(\kappa^{(\pi)} - \kappa^{(r)} - \mathsf{u}\left(\mathbf{e}_c + \mathbf{e}_\ell\right),\, \kappa^{(\pi)} - \mathsf{u}\mathbf{e}_c - \mathbf{e}_\ell\right) \right.\right.$$
$$\left. + \quad \mathcal{B}_i^{\mathbb{Q}}\left(\kappa^{(\pi)} - \kappa^{(r)} - \mathsf{u}\left(\mathbf{e}_c + \mathbf{e}_\ell\right),\, \kappa^{(\pi)} - \mathsf{u}\mathbf{e}_c - \mathbf{e}_\ell\right)' w_t \right\}$$
$$\left. - \exp\left\{ \mathcal{A}_i^{\mathbb{Q}}\left(\kappa^{(\pi)} - \kappa^{(r)} - \mathsf{u}\left(\mathbf{e}_c + \mathbf{e}_\ell\right),\, \kappa^{(\pi)} - \mathsf{u}\left(\mathbf{e}_c + \mathbf{e}_\ell\right)\right) + \mathcal{B}_i^{\mathbb{Q}}\left(\kappa^{(\pi)} - \kappa^{(r)} - \mathsf{u}\left(\mathbf{e}_c + \mathbf{e}_\ell\right),\, \kappa^{(\pi)} - \mathsf{u}\left(\mathbf{e}_c + \mathbf{e}_\ell\right)\right)' w_t \right\} \right]$$
$$+ \exp\left\{ n\left(\kappa_0^{(\pi)} - \kappa_0^{(r)}\right) + \mathcal{A}_n^{\mathbb{Q}}\left(\kappa^{(\pi)} - \kappa^{(r)} - \mathsf{u}\left(\mathbf{e}_c + \mathbf{e}_\ell\right),\, \kappa^{(\pi)} - \mathsf{u}\left(\mathbf{e}_c + \mathbf{e}_\ell\right)\right) \right.$$
$$\left. + \quad \left[\mathcal{B}_n^{\mathbb{Q}}\left(\kappa^{(\pi)} - \kappa^{(r)} - \mathsf{u}\left(\mathbf{e}_c + \mathbf{e}_\ell\right),\, \kappa^{(\pi)} - \mathsf{u}\left(\mathbf{e}_c + \mathbf{e}_\ell\right)\right) - \kappa^{(r)}\right]' w_t \right\}, \quad (23)$$

where $\mathbf{e}_\ell$ is such that $\delta_t^{(\ell)} = \mathbf{e}_\ell' w_t$. Despite the apparent complexity of these pricing formulas, note that they are weigthed sums of exponential-affine combinations of w_t and are hence computable easily through closed-form recursions. However, contrary to riskless term structures, the sovereign nominal and TIPS yields will not be affine functions of the factors, but rather non-linear combinations.

Last, we can easily obtain the BEI pricing formula by considering the log-difference of TIPS and nominal bond prices:

$$\mathrm{BEI}_t^{(n)} = \frac{1}{n}\left(\log B_t^{*(n)} - \log B_t^{(n)}\right). \tag{24}$$

4.8 The term structure of CDS

To identify sovereign credit risk, we consider the pricing of sovereign CDSs. Following our description of the CDS market in the previous sections, we assume that the protection seller delivers the nominal face value of any sovereign bond, irrespective of its nominal or inflation-protected nature, against the physical delivery of the cheapest-to-deliver bond.

Given our recovery assumptions, the cheapest-to-deliver in case of default is always the reference bond. Indeed, whatever the indexation honored by the sovereign government ρ^*, TIPS always deliver at least the same amount as nominal bonds in case of a credit event. Thus, the payment provided by the CDS is exactly equal to the loss given default of the nominal bonds, i.e. $1-\mathcal{P}_{\tau_c}^{(n_r)}$. The present value of the protection sold is given by:

$$\mathrm{PS}_t^{(n)} = \sum_{i=1}^{n} \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\sum_{j=0}^{i-1} r_{t+j}\right)\left(1-\mathcal{P}_{t+i}^{(n_r)}\right)\left(\mathbb{1}\left\{\sum_{j=0}^{i-1}\delta_{t+j}^{(c)}=0\right\} - \mathbb{1}\left\{\sum_{j=0}^{i}\delta_{t+j}^{(c)}=0\right\}\right)\right].$$

We assume that a buyer of protection makes periodic payments from time t to maturity n to protect against any type of credit event. The cash flow payment at time $t+i$ conditional on no default is designated as $\mathcal{S}_t^{(n)}$. The present value of the stream of cash flows paid by the protection buyer is:

$$\mathrm{PB}_t^{(n)} = \mathcal{S}_t^{(n)} \sum_{i=1}^{n} \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\sum_{j=0}^{i-1} r_{t+j}\right)\mathbb{1}\left\{\sum_{j=0}^{i}\delta_{t+j}^{(c)}=0\right\}\right]$$

No arbitrage pricing requires that the present value of the protection bought is equal to the present value of the protection sold. Equating both legs at inception, using the risk-neutral dynamics of Section 4.2 and solving for the swap spread yields:

$$\mathcal{S}_t^{(n)} = \frac{B_t^{(n)}\left[\mathrm{RR}_{\tau_c}=1\right] - B_t^{(n)}\left[\mathrm{RR}_{\tau_c}=\mathcal{P}_{\tau_c}^{(n_r)}\right]}{\sum_{i=1}^{n}\exp\left\{-i\kappa_0^{(r)} + \mathcal{A}_i^{\mathbb{Q}}\left(-\kappa^{(r)}-\mathsf{ue}_c,\ ,-\mathsf{ue}_c\right) + \left[\mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)}-\mathsf{ue}_c,\ ,-\mathsf{ue}_c\right)-\kappa^{(r)}\right]' w_t\right\}}, \tag{25}$$

where the notation $B_t^{(n)}\left[\mathrm{RR}_{\tau_c}=1\right]$ represents the price of a nominal bond for a recovery rate of 100%, and $B_t^{(n)}\left[\text{RR}_{\tau_c}=\mathcal{P}_{\tau_c}^{(n_r)}\right]$ is the exact pricing formula presented in Equation (22) (see Appendix A.7).

5 Data and Estimation

5.1 Data and identification

Let us first describe our empirical targets. All of our data is selected at the monthly frequency from November 2004 to October 2015, for data availability reasons. We select the 6m overnight-indexed swap (OIS) to guide our estimation for short-term riskless nominal yields, and select inflation-linked swap yields at maturities 1y, 2y, 3y, 5y, 7y, and 10y. Risky bonds are represented by GSW nominal Treasury yields at maturities 1y, 2y, 3y, 5y, 7y, and 10y, and by GSW breakeven inflation rates based on Treasuries of maturities 2y, 3y, 5y, 7y, and 10y. Instead of fitting the breakevens, we fit the ILS-BEI spreads, our main object of interest. We identify credit risk with the term structure of U.S. sovereign CDSs of maturities 2y, 3y, 5y, 7y, and 10y. Because all CDSs are virtually constant in the beginning of our sample, we consider their observations before 2008 as missing data. The liquidity intensity is identified by assuming that it relates linearly to the series of TIPS fitting errors obtained from a Nelson-Siegel-Svensson model.[22] We add monthly inflation data computed as the log-change of the CPI-U index provided by the Bureau of Labor Statistics (BLS). Lastly, we impose that $\delta_t=\mathbf{0}$ in the estimation procedure, thus δ_t are also part of the observable variables. We end up gathering 27 observable variables for each date denoted by $\mathcal{Y}_t\in\mathbb{R}^{27}$ that are all measured with error except inflation.

Using the state dynamics of Equation (10), the model can be expressed in state-space form where the measurement equations are given by:

$$\mathcal{Y}_t=\mathsf{F}\left(w_t,\,\vartheta^{\mathbb{Q}}\right)+\eta_t\,, \tag{26}$$

where η_t is a vector of i.i.d. Gaussian measurement errors with mean zero, $\mathsf{F}\left(w_t,\,\vartheta^{\mathbb{Q}}\right)$ is a non-linear function of the state, and $\vartheta^{\mathbb{Q}}$ represents the set of risk-neutral parameters driving the pricing equations. Since some of the pricing equations are non-linear, we estimate the model with the Extended Kalman Filter (EKF). The closed-form gradients of the pricing equations used for filtering are detailed in Appendix A.8.

In our estimation, we consider $N_x=3$, $N_{y_c}=2$ and $N_{y_\ell}=1$. Our latent factors are identified

[22] We are grateful to Richard Crump for providing the data.

through the following constraints. First, we impose that $\mu = \mathbf{0}$, Φ is lower-triangular, and $\Sigma = I_{N_x}$ such that x_t cannot be rotated. Second, we fix β_y as lower-triangular such that credit factors Granger-cause liquidity factors, consistent with the intuition. Diagonals of both Φ and $\text{diag}(\mathsf{c})\beta$ are in the unit circle to ensure stationarity. For the scaling of y_t, we set $\mathsf{c}_y^{\mathbb{Q}} = \mathbf{1}$. For parsimony, we impose that the covariance matrix of measurement errors to be diagonal, and each block of observables have a different standard deviation parameter. We set the standard deviation of the liquidity proxy measurement errors to a fifth of its in-sample standard deviation, as it provides a reasonable fit of the proxy. Last, some parameters are technically identified but the log-likelihood is nearly flat close to their estimates. A first round of estimation shows that this is the case for c_δ for the credit-event variable, which we set to 0.6 (see Monfort et al. (2020)), and for the indexation parameter ρ^*. We set it to 0 in the estimation, noting that switching it to 1 barely changes the remaining estimates after estimation. We end up with a total of 52 parameters.

5.2 Parameter estimates

The estimates obtained from our maximum likelihood procedure are reported in Table X. All parameters are significant at the 5% level. As expected, all processes are extremely persistent under both the physical and the risk-neutral measure. It is worth analyzing the parameters of the inflation equation. First, we obtain a negative correlation between the credit-event intensity factors $y_t^{(c)}$ and inflation, with parameters $\kappa_y^{(\pi)} = (-0.1915, -0.0022)'$ annualized. Therefore, inflation tends to go down when the probability of a sovereign default goes up, consistent with the increased risk of deflation happening during the financial crisis. Second, our model estimates favor the existence of hyperinflation upon default, with a feedback of the credit-event variable on inflation at $\kappa_\delta^{(\pi)} = 0.3138$. Economically, this estimate translates into an average monthly inflation jump of 18.8 percentage points, about 40 times its standard deviation during the sample.

The six latent factors, x_t and y_t, filtered by our model are plotted in Figure III. The first three factors x_t are presented on panel (a) and control the term structure of discount rates as well as part of the inflation dynamics. Panel (b) presents the three components of the factors y_t, controlling the default and liquidity-event probabilities. In panels (b.1) and (b.2), we see that the credit factors allow us to perfectly track CDS spreads, reproducing the large spike observed during the financial crisis and staying elevated afterwards. In contrast, the liquidity factor experiences one large spike in 2008 and dies out quickly afterwards.

5.3 Model performance

We now turn to the fitting properties of the model. We present RMSEs and R-squared measures in Table XI. The model does a tremendous job in capturing the bulk of fluctuations of the four different term structures and the monthly inflation rate with only 6 factors. All RMSEs, besides OIS, are between 2bps and 16.5bps, with ranges of [10.5bps-16.5bps] for the ILS term structure, [5.4bps-11.3bps] for the nominal term structure, [7.6bps-14bps] for the ILS-BEI term structure, and [2.6bps-4.7bps] for the CDS term structure. As a result, many R-squared are above 98%, and most of them are well above 80%. By construction, the R-squared on inflation is 1, since we imposed that inflation is measured without errors. Note that only the OIS shows a slightly worse performance, since we did not impose consistency of the model with the ZLB, a crucial feature for reproducing the short-end of the curve.

We also present the time series of the fitted values produced by the model for the OIS yield, the ILS term structure, the nominal Treasury term structure, the ILS-BEI spreads, and the CDS term structure on Figures IV, V, VI, VII, and VIII, respectively. Fitted values are virtually indistinguishable from the observed data for most of the observables, confirming the outstanding fitting performance of our term structure framework. We are thus confident that our model is successful in identifying the different pricing components, namely credit and liquidity, contained in the different observable variables that we used as inputs.

5.4 Decomposition of ILS-BEI Spreads

To understand the relative importance of default risk in driving a wedge between ILS and BEI, we fit the term structure of ILS-BEI spreads by employing only the credit risk factors $y_t^{(c)}$. The results are plotted in Figure IX.

Three observations can be discerned from Figure IX. First, the credit component of the ILS-BEI spread contributes nearly completely to the overall fit of the curves prior to 2008. This is because we have considered the CDS term structure as missing data before the financial crisis, as seen in Figure VIII. As a result, the filter interprets our credit factors as essentially unconstrained by the CDSs, and it uses the flexibility of these factors to concentrate on fitting the ILS-BEI spreads. Second, in the middle of the crisis around September of 2008, the peak of the ILS-BEI spread is mostly driven by the liqudity factor. This is not surprising either since the peaks of the liquidity proxy and the peak observed on the ILS-BEI spreads coincide. In contrast, the credit component remains low until it peaks in 2009. Third, the credit component of the ILS-BEI spread is the dominant factor in capturing the variability of the ILS-BEI curves in the data in the post-crisis period. The credit component represents between 20bps and 40bps of the ILS-BEI spreads in the post crisis period, depending on the time and maturity.

The credit component does not show strong term structure effects, its absolute magnitude being roughly constant with respect to maturity. However, since the ILS-BEI mean and volatility compresses with maturity, the relative importance of the credit component grows with maturity. We plot the importance of credit risk as a ratio of the ILS-BEI spreads on Figure X. We see that the proportion explained by credit risk indeed increases with maturity, especially during the 2010-2014 period where it represent about 60% of the 2y spread against about 90% of the 10y spread. Although credit risk goes down at the end of our sample, it remains an important driver of the ILS-BEI spreads.

Our asset pricing model validates the results of our panel regressions in Section 3.2. In particular, over the full sample between 2008 and 2015, the U.S. CDS spread driven by the credit factors has positive and significant explanatory power of the ILS-BEI spreads after controlling for liquidity. Moreover, contrasting Column (7) in Tables V and VI, we see that the explanatory power of the credit component (proxied by the CDS spread) is statistically weaker (t-statistic of 2.16) during the crisis period and much stronger (t-statistic of 4.46) in the post-crisis period. Whereas the opposite is true for the liquidity factor (proxied by the OTR Difference) with t-statistic of 2.66 in Table V and t-statistic of 0.57 in Table VI. This is consistent with the decomposition of the fitted ILS-BEI curves shown in Figures IX and X.

5.5 Credit risk premium and mispricing

Our estimated term structure model also allows us to perform a risk-premium decomposition of the ILS-BEI. We thus ask how much of the credit component and the total risk premium charged on ILS-BEI is the result of investors charging a differential credit risk premium to nominal bonds and TIPS. To obtain risk premia components, we recompute counterfactual ILS-BEI spreads under the expectation hypothesis (i.e. risk-neutral parameters are set to the corresponding physical parameter estimates), and subtract them from the observed ILS-BEI. This provides us the total risk premia on the spreads. Similarly, we compute the counterfactual expectation hypothesis spreads setting the liquidity factor to zero, and subtract them from the credit components to obtain credit risk premia.

Figure XI and XII present the credit risk premia contained in ILS-BEI spreads along with the total risk premia and the credit component, respectively. Two main conclusions emerge from the figures. First, credit risk premia represent about a quarter to a third of both the total premia and the credit component, between 10bps and 20bps. Second, the credit premia is highly correlated with the credit component contained in ILS-BEI spreads so they both follow the same factor structure. This shows that investors risk premia tend to grow hand in hand with the differential exposure of nominal Treasuries and TIPS. This seems to be consistent with the outstanding share of TIPS being roughly constant at 10% of the total outstanding U.S. debt.

6 Conclusion

In this paper, we explore the relative pricing of nominal and real U.S. sovereign securities in the presence of credit risk. We argue that while most of the previous studies attribute the mispricing of TIPS to liquidity factors or slow moving capital, credit risk can also represent a significant driver of deviations oftentimes interpreted as violations of no-arbitrage. Our study shows that in the presence of credit risk, the spreads between inflation-linked swaps and breakeven inflation rates reflect differences in the propensity of the sovereign to reimburse nominal and real bonds in case of default. We hypothesize this result is driven by a difference in recovery rates. Our empirical approach shows U.S. CDS spreads are positively correlated with first differences in ILS-BEI spreads after the financial crisis, while controlling for liquidity and potential alternative explanations. We then conduct a more formal empirical analysis through an intensity-based affine asset pricing model. We show that credit risk factors extracted from the CDS are able to explain most of the ILS-BEI yield curve after the financial crisis. Our model estimates confirm the existence of a lower recovery rate for TIPS than for nominal bonds by about 8 percentage points.

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Tables

Table I: **Summary Statistics**

Table I provides summary statistics for the variables used in the regression analysis. Panel A includes the full sample period from January 2008 to October 2015. Panel B is the post crisis subsample from January 2010 to October 2015. $ILS - BEI$ is the difference in the 5-year inflation swap rate and the 5-year breakeven inflation rate (Treasury-TIPS). Both $Tsy\ ZC\ Yield$ and $TIPS\ ZC\ Yield$ are for the 5-year maturity. 5-year $US\ CDS$ spreads are denominated in EUR. $LIBOR - OIS$ is the difference in the London Inter-bank Offered Rate and the overnight indexed swap rate. $HPWNoise$ follows Hu, Pan and Wang (2013). $TIPS\ Noise$ measures average daily deviations in the real yield curve. VIX denotes the CBOE Volatility Index.

Panel A			**Full Sample**		
	Mean	SD	Min	Max	N
ILS-BEI (bps)	36	30	-1	210	1902
Infl Swap Rate	2.04	0.49	-0.57	3.31	1902
Tsy ZC Yield	1.74	0.70	0.59	3.76	1902
TIPS ZC Yield	0.06	1.04	-1.72	3.88	1902
US CDS (bps)	33	16	6	100	1902
LIBOR-OIS	0.34	0.43	0.06	3.64	1902
HPW Noise	3.51	3.50	0.72	20.47	1902
TIPS Noise	5.93	5.06	2.05	41.8	1902
VIX	21.96	10.44	10.32	80.86	1902
Panel B			**Post Crisis**		
	Mean	SD	Min	Max	N
ILS-BEI (bps)	23	10	-1	59	1403
Infl Swap Rate	2.09	0.29	1.24	2.71	1403
Tsy ZC Yield	1.45	0.51	0.59	2.79	1403
TIPS ZC Yield	-0.41	0.62	-1.72	0.83	1403
US CDS (bps)	34	12	14	63	1403
LIBOR-OIS	0.19	0.09	0.06	0.50	1403
HPW Noise	1.99	0.74	0.72	4.58	1403
TIPS Noise	4.72	1.41	2.05	7.58	1403
VIX	18.40	6.13	10.32	48.00	1403

Table II: **ILS-BEI - January 2008 to October 2015**

Table II shows the results from a panel regression of ILS-BEI on US CDS spreads and various controls using daily observations. The sample period is from January 2008 to October 2015. $ILS - BEI$ is the difference in the inflation swap rate and the breakeven inflation rate (Treasury-TIPS) for 2-, 3-. 5-, 7-, and 10-year tenors. $US\ CDS$ spreads are for the 5-year tenor. $HPW Noise$ follows Hu, Pan and Wang (2013). $TIPS\ Noise$ measures average daily deviations in the real yield curve. $LIBOR - OIS$ is the difference in the London Inter-bank Offered Rate and the overnight indexed swap rate. $OTR\ Difference$ is the difference in 10-year Treasury par yield from Gurkaynak, Sack and Wright (2006) less the on-the-run 10-year Treasury yield from Bloomberg. VIX denotes the CBOE Volatility Index. Standard errors are reported in parentheses.

Dep Var: ILS-BEI Spread	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
US CDS	0.196***	0.195***	0.218***	0.205***	0.186***	0.218***	0.228***	0.228***
	(0.049)	(0.049)	(0.049)	(0.050)	(0.049)	(0.051)	(0.051)	(0.051)
ILS-BEI_{t-1}	0.822***	0.822***	0.823***	0.822***	0.822***	0.822***	0.823***	0.817***
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)
HPW Noise		0.111					0.637**	0.633**
		(0.283)					(0.291)	(0.290)
TIPS Noise			−1.077***				−1.039***	−1.033***
			(0.172)				(0.182)	(0.182)
LIBOR-OIS				−3.303*			−3.965**	−3.952**
				(1.826)			(1.924)	(1.921)
OTR Difference					−24.810***		−17.395***	−17.457***
					(4.816)		(5.086)	(5.076)
VIX						−0.065*	−0.023	−0.022
						(0.039)	(0.041)	(0.041)
Week	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Tenor	No	No	No	No	No	No	No	Yes
Observations	9147	9147	9147	9147	9147	9127	9127	9127
$1 - \mathbb{V}(\varepsilon_t)/\mathbb{V}[\Delta(\text{ILS}_t - \text{BEI}_t)]$	0.148	0.148	0.152	0.149	0.151	0.149	0.155	0.158

*,**,*** represent statistical significance at the 10%, 5%, and 1% critical threshold, respectively.

Table III: **First Differences of ILS-BEI - January 2008 to October 2015**

Table III shows the results from a panel regression of the change in ILS-BEI on the change in US CDS spreads and various controls using daily observations. The sample period is from January 2008 to October 2015. $ILS-BEI$ is the difference in the inflation swap rate and the breakeven inflation rate (Treasury-TIPS) for 2-, 3-. 5-, 7-, and 10-year tenors. $US\ CDS$ spreads are for the 5-year tenor. $HPWNoise$ follows Hu, Pan and Wang (2013). $TIPS\ Noise$ measures average daily deviations in the real yield curve. $LIBOR-OIS$ is the difference in the London Inter-bank Offered Rate and the overnight indexed swap rate. $OTR\ Difference$ is the difference in 10-year Treasury par yield from Gurkaynak, Sack and Wright (2006) less the on-the-run 10-year Treasury yield from Bloomberg. VIX denotes the CBOE Volatility Index. Standard errors are reported in parentheses.

Dep Var: Δ ILS-BEI Spread	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Δ US CDS	0.154***	0.153***	0.161***	0.161***	0.141***	0.168***	0.160***	0.160***
	(0.052)	(0.052)	(0.052)	(0.052)	(0.052)	(0.052)	(0.052)	(0.052)
Δ HPW Noise		−0.402					−0.241	−0.241
		(0.254)					(0.256)	(0.256)
Δ TIPS Noise			−1.872***				−1.806***	−1.807***
			(0.205)				(0.207)	(0.207)
Δ LIBOR-OIS				−7.768***			−3.910	−3.910
				(2.713)			(2.835)	(2.836)
Δ OTR Difference					−21.738***		−16.946***	−16.946***
					(3.456)		(3.695)	(3.696)
Δ VIX						−0.097***	−0.040	−0.040
						(0.037)	(0.038)	(0.038)
Week	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Tenor	No	No	No	No	No	No	No	Yes
Observations	9142	9142	9137	9142	9142	9107	9102	9102
R^2	0.001	0.001	0.006	0.001	0.005	0.001	0.009	0.053

*,**,*** represent statistical significance at the 10%, 5%, and 1% critical threshold, respectively.

Table IV: **Components of ILS-BEI Spread - January 2008 to October 2015**

Table IV shows the results from a panel regression of TIPS yields, Treasury yields, and ILS spreads on US CDS spreads and various controls using daily observations. The sample period is from January 2008 to October 2015. $US\ CDS$ spreads are for the 5-year tenor. $HPW Noise$ follows Hu, Pan and Wang (2013). $TIPS\ Noise$ measures average daily deviations in the real yield curve. $LIBOR - OIS$ is the difference in the London Inter-bank Offered Rate and the overnight indexed swap rate. $OTR\ Difference$ is the difference in 10-year Treasury par yield from Gurkaynak, Sack and Wright (2006) less the on-the-run 10-year Treasury yield from Bloomberg. VIX denotes the CBOE Volatility Index. Standard errors are reported in parentheses.

Dep Var:	(1) TIPS	(2) Nominal	(3) ILS
US CDS	0.268^{***}	0.123^{***}	0.061
	(0.047)	(0.045)	(0.061)
TIPS_{t-1}	0.985^{***}		
	(0.002)		
Nominal $_{t-1}$		0.972^{***}	
		(0.002)	
ILS $_{t-1}$			0.960^{***}
			(0.003)
HPW Noise	-0.735^{***}	-1.910^{***}	-0.489
	(0.266)	(0.254)	(0.345)
TIPS Noise	-1.750^{***}	-0.009	0.541^{**}
	(0.167)	(0.159)	(0.216)
LIBOR-OIS	-28.185^{***}	-20.507^{***}	3.139
	(1.762)	(1.684)	(2.281)
OTR Difference	-19.114^{***}	-22.629^{***}	-20.587^{***}
	(4.651)	(4.449)	(6.026)
VIX	0.077^{**}	-0.454^{***}	-0.568^{***}
	(0.037)	(0.036)	(0.048)
Week	Yes	Yes	Yes
Tenor	Yes	Yes	Yes
Observations	9130	9130	9130
$1 - \mathbb{V}(\varepsilon_t)/\mathbb{V}[\Delta(\text{ILS}_t - \text{BEI}_t)]$	0.290	0.249	0.176

*,**,*** represent statistical significance at the 10%, 5%, and 1% critical threshold, respectively.

Table V: **ILS-BEI - January 2008 to December 2009**

Table V shows the results from a panel regression of ILS-BEI on US CDS spreads and various controls using daily observations. The sample period is from January 2008 through December 2009. $ILS - BEI$ is the difference in the inflation swap rate and the breakeven inflation rate (Treasury-TIPS) for 2-, 3-. 5-, 7-, and 10-year tenors. $US\ CDS$ spreads are for the 5-year tenor. $HPWNoise$ follows Hu, Pan and Wang (2013). $TIPS\ Noise$ measures average daily deviations in the real yield curve. $LIBOR - OIS$ is the difference in the London Inter-bank Offered Rate and the overnight indexed swap rate. $OTR\ Difference$ is the difference in the on-the-run 10-year U.S. Treasury and the off-the-run 9-year U.S. Treasury from the Bloomberg on/off-the-run U.S. Treasury curve. VIX denotes the CBOE Volatility Index. Standard errors are reported in parentheses.

Dep Var: ILS-BEI Spread	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
US CDS	0.213*	0.209*	0.258**	0.232*	0.199*	0.258**	0.270**	0.264**
	(0.120)	(0.121)	(0.121)	(0.122)	(0.120)	(0.124)	(0.125)	(0.121)
ILS-BEI$_{t-1}$	0.795***	0.795***	0.796***	0.795***	0.795***	0.796***	0.797***	0.698***
	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.013)
HPW Noise		0.364					1.246*	1.091*
		(0.631)					(0.660)	(0.636)
TIPS Noise			−1.139***				−1.011***	−0.886***
			(0.321)				(0.354)	(0.341)
LIBOR-OIS				−3.374			−3.272	−3.001
				(3.256)			(3.594)	(3.462)
OTR Difference					−44.458***		−32.578***	−34.241***
					(11.188)		(12.240)	(11.792)
VIX						−0.151	−0.072	−0.055
						(0.098)	(0.106)	(0.102)
Week	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Tenor	No	No	No	No	No	No	No	Yes
Observations	2387	2387	2387	2387	2387	2387	2387	2387
$1 - \mathbb{V}(\varepsilon_t)/\mathbb{V}[\Delta(\text{ILS}_t - \text{BEI}_t)]$	0.168	0.168	0.172	0.168	0.173	0.168	0.177	0.238

*,**,*** represent statistical significance at the 10%, 5%, and 1% critical threshold, respectively.

Table VI: **ILS-BEI - January 2010 to October 2015**

Table VI shows the results from a panel regression of ILS-BEI on US CDS spreads and various controls using daily observations. The sample period is from January 2010 to October 2015. $ILS - BEI$ is the difference in the inflation swap rate and the breakeven inflation rate (Treasury-TIPS) for 2-, 3-. 5-, 7-, and 10-year tenors. $US\ CDS$ spreads are for the 5-year tenor. $HPWNoise$ follows Hu, Pan and Wang (2013). $TIPS\ Noise$ measures average daily deviations in the real yield curve. $TIPSNoise$ measures average daily deviations in the real yield curve. $LIBOR - OIS$ is the difference in the London Inter-bank Offered Rate and the overnight indexed swap rate. $OTR\ Difference$ is the difference in 10-year Treasury par yield from Gurkaynak, Sack and Wright (2006) less the on-the-run 10-year Treasury yield from Bloomberg. VIX denotes the CBOE Volatility Index. Standard errors are reported in parentheses.

Dep Var: ILS-BEI Spread	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
US CDS	0.171***	0.170***	0.172***	0.171***	0.172***	0.164***	0.165***	0.167***
	(0.035)	(0.035)	(0.035)	(0.035)	(0.035)	(0.037)	(0.037)	(0.037)
ILS-BEI_{t-1}	0.905***	0.905***	0.905***	0.905***	0.905***	0.904***	0.904***	0.888***
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.006)
HPW Noise		−0.396*					−0.381*	−0.376
		(0.230)					(0.231)	(0.230)
TIPS Noise			−0.415				−0.377	−0.375
			(0.268)				(0.270)	(0.269)
LIBOR-OIS				−4.591			−3.509	−3.280
				(7.704)			(7.769)	(7.739)
OTR Difference					1.563		2.120	2.056
					(3.667)		(3.737)	(3.723)
VIX						0.022	0.018	0.020
						(0.028)	(0.028)	(0.028)
Week	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Tenor	No	No	No	No	No	No	No	Yes
Observations	6755	6755	6755	6755	6755	6735	6735	6735
$1 - \mathbb{V}(\varepsilon_t)/\mathbb{V}[\Delta(\text{ILS}_t - \text{BEI}_t)]$	0.100	0.101	0.101	0.101	0.101	0.101	0.102	0.109

*,**,*** represent statistical significance at the 10%, 5%, and 1% critical threshold, respectively.

Table VII: **Liquidity Premia and CDS**

Table VII presents results of an analysis of liquidity premia. In Panel A, we present results from regressions

$$\begin{aligned} BEI_t &= a_1 + a_2 OTR_t + a_3 VOL_t + a_4 ILS - BEI_t + a_5 CPI_t^e + a_6 CFNAI_t + \epsilon_{1t} \\ BEI_t &= b_1 + b_2 OTR_t + b_3 VOL_t + b_4 ILS - BEI_t + b_5 CPI_t^e + b_6 CFNAI_t + b_7 US\ CDS_t + \epsilon_{2t}, \end{aligned}$$

where the dependent variable is breakeven inflation, and the independent variables are OTR, the on-the-run 10-Year Treasury Spread, VOL, the log ratio of volume in the TIPS market to the nominal Treasury market, $ILS - BEI$, the inflation swap-adjusted BEI, CPI^e, the median forecast of 10-year CPI inflation from the Survey of Professional Forecasters, $CFNAI$, the Chicago Fed National Activity Index, and $US\ CDS$, the 5-year credit default swap spread for U.S. Treasury securities. In Panel B, the estimated liquidity premium from Panel A is regressed on the U.S. CDS spread. The liquidity premium is measured as

$$\hat{L}_t = -\left(\hat{a}_2 OTR_t + \hat{a}_3 VOL_t + \hat{a}_4 ILS - BEI_t\right).$$

Newey-West standard errors are reported in parentheses.

Panel A: Breakeven Inflation

Dep Var: BEI	(1)	(2)
OTR	-1.143^{**}	-1.218^{***}
	(0.153)	(0.155)
VOL	-0.438^{***}	-0.472^{***}
	(0.058)	(0.060)
$ILS - BEI$	-1.284^{***}	-1.158^{***}
	(0.103)	(0.109)
CPI^e	0.960^{***}	0.997^{***}
	(0.109)	(0.106)
$CFNAI$	0.027^{*}	0.031^{**}
	(0.014)	(0.014)
$US\ CDS$		-0.214^{***}
		(0.075)
R^2	0.651	0.656

Panel B: Liquidity Premium

Dep Var: $\hat{L}$	
$US\ CDS$	0.608^{***}
	(0.126)
R^2	0.086

Notes: *,**,*** represent statistical significance at the 10%, 5%, and 1% critical threshold, respectively.

Table VIII: **United Kingdom ILS-BEI Spread - January 2010 to October 2015**

Table VIII shows the results from a panel regression of the change in UK ILS-BEI spreads on the changes in UK CDS spreads and various controls using daily observations. The sample period is from January 2010 to October 2015. $UK\ CDS$ spreads are for the 5-year tenor. $LIBOR - OIS$ is the difference in the GBP London Inter-bank Offered Rate and the overnight indexed swap rate. $OTR\ Difference$ is the difference in 10-year yield from the Bank of England website less the on-the-run 10-year gilt yield from Bloomberg. $VSTOXX$ denotes the EURO STOXX 50 Volatility Index. Standard errors are reported in parentheses.

Dep Var: UK ILS-BEI Spread	(1)	(2)	(3)	(4)	(5)	(6)
UK CDS	0.112***	0.106***	0.116***	0.074***	0.072***	0.072***
	(0.024)	(0.024)	(0.024)	(0.027)	(0.027)	(0.027)
UK ILS-BEI$_{t-1}$	0.966***	0.966***	0.966***	0.965***	0.965***	0.953***
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.004)
OTR Difference		−18.497***			−18.605***	−18.866***
		(4.032)			(4.044)	(4.033)
GBP LIBOR-OIS			−20.424***		−20.543***	−20.000**
			(7.885)		(7.915)	(7.895)
VSTOXX				0.097***	0.102***	0.103***
				(0.032)	(0.032)	(0.032)
Week	Yes	Yes	Yes	Yes	Yes	Yes
Tenor	No	No	No	No	No	Yes
Observations	6482	6482	6482	6427	6427	6427
$1 - \mathbb{V}(\varepsilon_t)/\mathbb{V}[\Delta(\text{ILS}_t - \text{BEI}_t)]$	0.137	0.140	0.138	0.140	0.144	0.149

*,**,*** represent statistical significance at the 10%, 5%, and 1% critical threshold, respectively.

Table IX: **First Difference of United Kingdom ILS-BEI Spread - January 2010 to October 2015**

Table IX shows the results from a panel regression of the change in UK ILS-BEI spreads on the changes in UK CDS spreads and various controls using daily observations. The sample period is from January 2010 to October 2015. $UK\ CDS$ spreads are for the 5-year tenor. $LIBOR - OIS$ is the difference in the GBP London Inter-bank Offered Rate and the overnight indexed swap rate. $OTR\ Difference$ is the difference in 10-year yield from the Bank of England website less the on-the-run 10-year gilt yield from Bloomberg. $VSTOXX$ denotes the EURO STOXX 50 Volatility Index. Standard errors are reported in parentheses.

Dep Var: Δ UK ILS-BEI Spread	(1)	(2)	(3)	(4)	(5)	(6)
Δ UK CDS	0.266***	0.267***	0.240***	0.218***	0.192***	0.192***
	(0.026)	(0.026)	(0.026)	(0.030)	(0.030)	(0.030)
Δ GBP LIBOR-OIS		−6.748			−7.602	−7.597
		(8.754)			(8.708)	(8.711)
Δ OTR Difference			−34.044***		−35.042***	−35.04***
			(3.209)		(3.226)	(3.227)
Δ VSTOXX				0.106***	0.108***	0.108***
				(0.030)	(0.030)	(0.030)
Week	Yes	Yes	Yes	Yes	Yes	Yes
Tenor	No	No	No	No	No	Yes
Observations	6366	6366	6366	6299	6299	6299
R^2	0.138	0.138	0.154	0.141	0.158	0.158

*,**,*** represent statistical significance at the 10%, 5%, and 1% critical threshold, respectively.

Table X: **Term structure model parameter Estimates**

Estimation is performed by extended Kalman filter-based maximum likelihood. For numerical stability, we impose that the time series of δ_t are fitted in the filter but treat them as unobservables as far as the likelihood is concerned. This avoids the filter to favor parameter estimates where the pricing of observables is down-weighted with respect to the dynamics of $\delta_t = 0$. Standard errors are of estimation are computed with outer-product approximation and are reported in brackets. When the parameter has been calibrated or the estimates hit the bound, we report '–' instead of the standard errors. Although not introduced in the main text, $\kappa_0^{(\ell)}$ and $\kappa^{(\ell)}$ are the parameters governing the liquidity proxy equation, such that the proxy is equal to $\kappa_0^{(\ell)} + \kappa^{(\ell)\prime}\, w_t$.

	μ	Φ				$1200 \cdot \kappa_x^{(r)}$	$1200 \cdot \kappa_x^{(\pi)}$	$\kappa_x^{(\ell)}$	$\mu^{\mathbb{Q}}$	$\Phi^{\mathbb{Q}}$		
x_t	0	0.99999	0	0		0.0950	1.2094	0	−0.1854	0.8151	−0.2330	0.2328
	–	–	–	–		[0.0451]	[0.2811]	–	[0.0396]	[0.0478]	[0.0616]	[0.0385]
	0	−0.00011	0.9991	0		0.4586	1.6251	0	−0.0894	−0.3711	0.5033	0.3461
	–	[0.00005]	[0.0018]	–		[0.0736]	[0.2193]	–	[0.0796]	[0.1020]	[0.0462]	[0.0732]
	0	0.1548	0.2086	0.8878		−0.1809	−1.3238	0	0.0039	−0.0215	−0.0270	1.0071
	–	[0.0374]	[0.0600]	[0.0549]		[0.0580]	[0.2041]	–	[0.00002]	[0.0077]	[0.0064]	[0.0137]
	ν	β			c_y	$\kappa_y^{(r)}$	$1200 \cdot \kappa_y^{(\pi)}$	$\kappa_y^{(\ell)}$	θ_y	$\beta^{\mathbb{Q}}$		
y_t	0.0904	0.9490	0	0	0.9912	0	−0.1915	0	0.0089	0.9574	0	0
	[0.0230]	[0.00001]	–	–	[0.0019]	–	[0.0058]	–	[0.0019]	[0.0018]	–	–
	0.0747	0.0031	0.99999	0	0.9921	0	−0.0022	0	0.0080	0.0032	1.0080	0
	[0.0119]	[0.0015]	–	–	[0.0001]	–	[0.0004]	–	[0.0001]	[0.0015]	[0.0001]	–
	0.0279	0.00007	0.0054	0.9776	0.9750	0	0	0.7226	0.0257	0.0001	0.0055	1.0027
	[0.0058]	[0.000005]	$[4 \cdot 10^{-8}]$	[0.0139]	[0.0125]	–	–	[0.0424]	[0.0131]	[0.000005]	[0.0021]	[0.0048]
		$1200 \cdot \beta_\lambda$			c_δ	$\kappa_\delta^{(r)}$	$\kappa_\delta^{(\pi)}$	$\kappa_\delta^{(\ell)}$				
$\delta_t^{(c)}$		0.0060	0.0407	0	0.6	0	0.3138	0				
		[0.0011]	[0.0066]	–	–	–	[0.0308]	–				
$\delta_t^{(\ell)}$		0	0	0.2265	0.3330	0	0	0				
		–	–	[0.0227]	[0.0366]	–	–	–				
	σ_{ois}	σ_{ILS}	σ_{nom}	$\sigma_{\text{ILS-BEI}}$	σ_{CDS}	$\kappa_0^{(r)}$	$\kappa_0^{(\pi)}$	$\kappa_0^{(\ell)}$				
	0.4621	0.1376	0.1149	4.0083	0.1025	0.0048	0.0033	2.5967				
	[0.0628]	[0.0041]	[0.0037]	[0.1386]	[0.0034]	[0.0020]	[0.0004]	[0.1529]				

Table XI: **Observable variables Root mean squared error and R-squared**

All RMSEs are in basis points while R^2 measures are in natural units. 'ILS' stands for inflation-linked swaps, 'Nominal' is the corresponding Treasury curve, 'ILS-BEI' is the spread between inflation-linked swaps and equivalent maturity Treasury breakevens, 'CDS' correspond to the U.S. sovereign CDS spreads, 'OIS' is the 6m overnight indexed swap rate, π_t is the monthly annualized inflation rate and 'Liq' is the liquidity proxy defined as the errors from a Nelson-Siegel-Svensson model applied on the TIPS individual bonds.

		1y	2y	3y	5y	7y	10y
ILS	RMSE (bps)	16.45	10.96	10.61	11.3	13.19	14.35
	R^2	(0.983)	(0.986)	(0.98)	(0.957)	(0.907)	(0.811)
Nominal	RMSE (bps)	9.64	7	10.08	8.83	5.4	11.32
	R^2	(0.997)	(0.998)	(0.996)	(0.996)	(0.998)	(0.988)
ILS-BEI	RMSE (bps)		14.04	11.03	10.53	8.1	7.6
	R^2		(0.833)	(0.864)	(0.843)	(0.829)	(0.718)
CDS	RMSE (bps)		4.71	3.01	3.47	2.63	3.93
	R^2		(0.833)	(0.939)	(0.94)	(0.964)	(0.913)
		OIS	π_t	Liq			
	RMSE (bps)	43	4.53	1.8			
	R^2	(0.953)	(1)	(0.867)			

Figures

Figure I: Five-year inflation-linked swap and breakeven inflation rate

This figure presents the ILS and BEI daily zero-coupon data for the 5-year maturity from 2005 to 2016 on panel (a). ILS data is taken from bloomberg while nominal and TIPS zero coupon yields are taken from GSW 2006 and 2010 database. Panel (b) presents the spread between ILS-BEI and the CDS spread on the same graph.

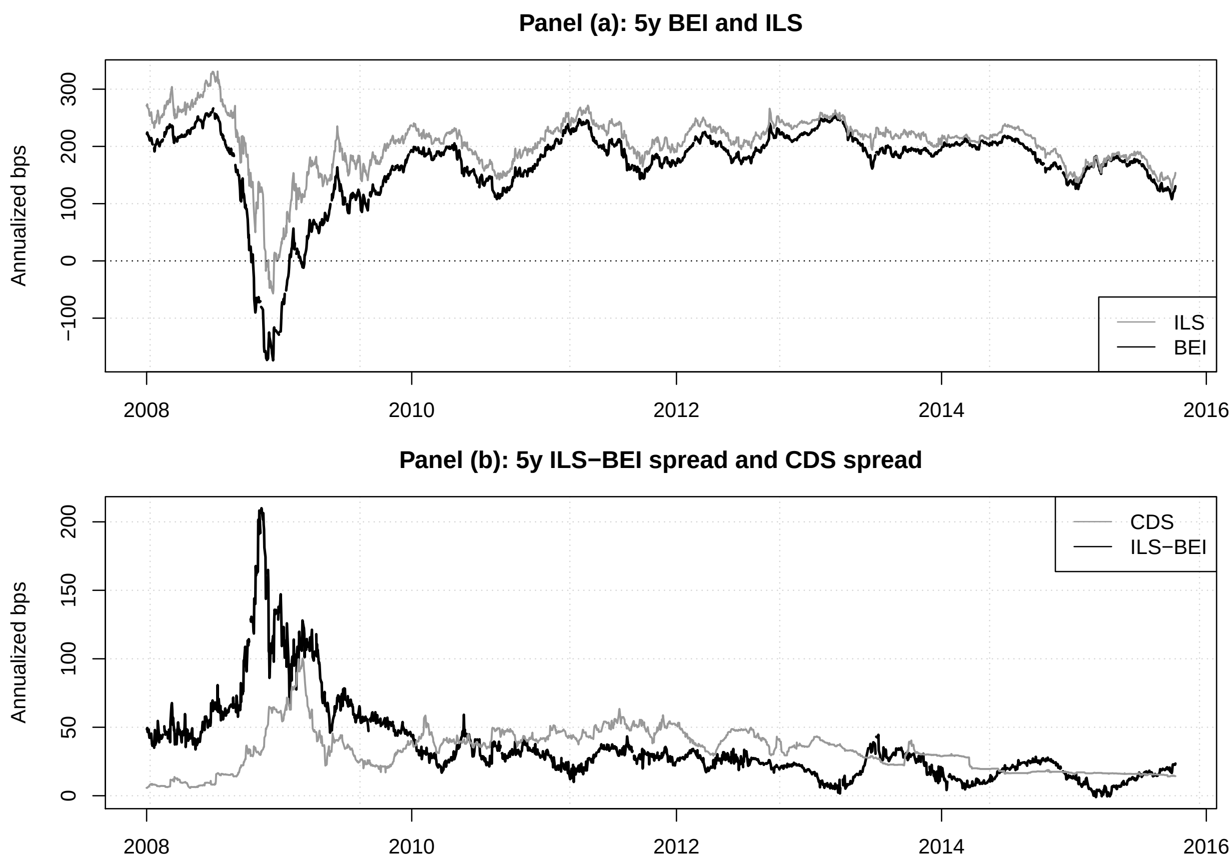


Figure II: Spread between zero-coupon inflation-linked swaps and breakevens

This plot presents daily data of the spread between zero-coupon inflation-linked swaps and breakevens of the corresponding maturity, from January 2008 to October 2015. ILS data is taken from Bloomberg while nominal and TIPS zero-coupon yields are taken from GSW 2006 and 2010 database. Maturities range from 2 years (black line) to 10 years (light grey line).

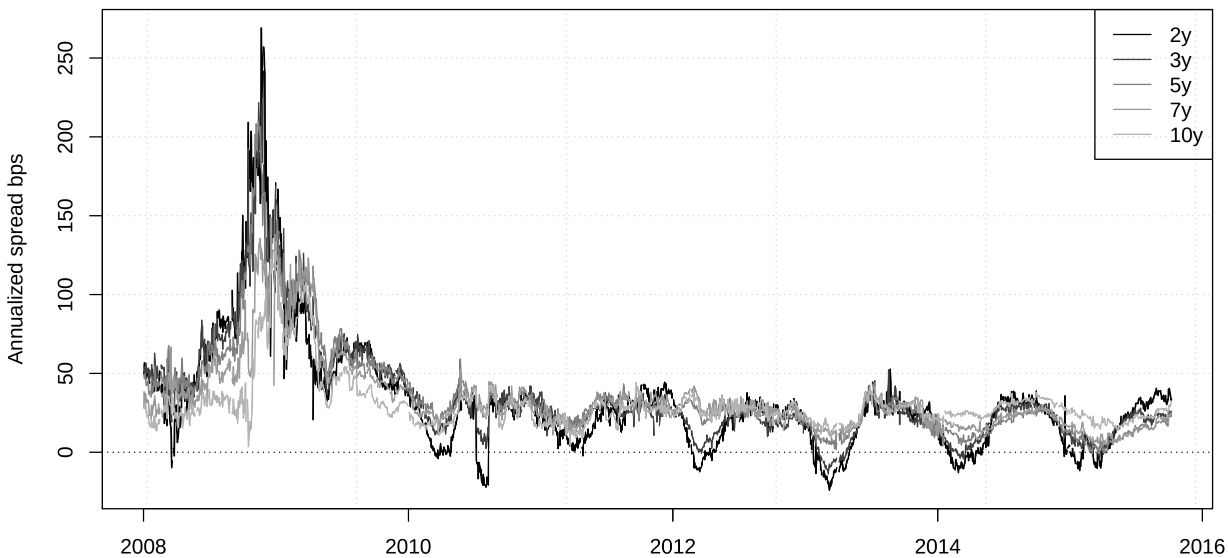


Figure III: Filtered Factors

Factors are estimated by extended Kalman filter. Data range from 2004 to 2015. Panels (a.1) to (a.3) present the three components of x_t while panels (b.1) to (b.3) present the three components of y_t. (b.1) and (b.2) correspond to the credit factors $y_t^{(c)}$ whereas (b.3) corresponds to $y_t^{(\ell)}$.

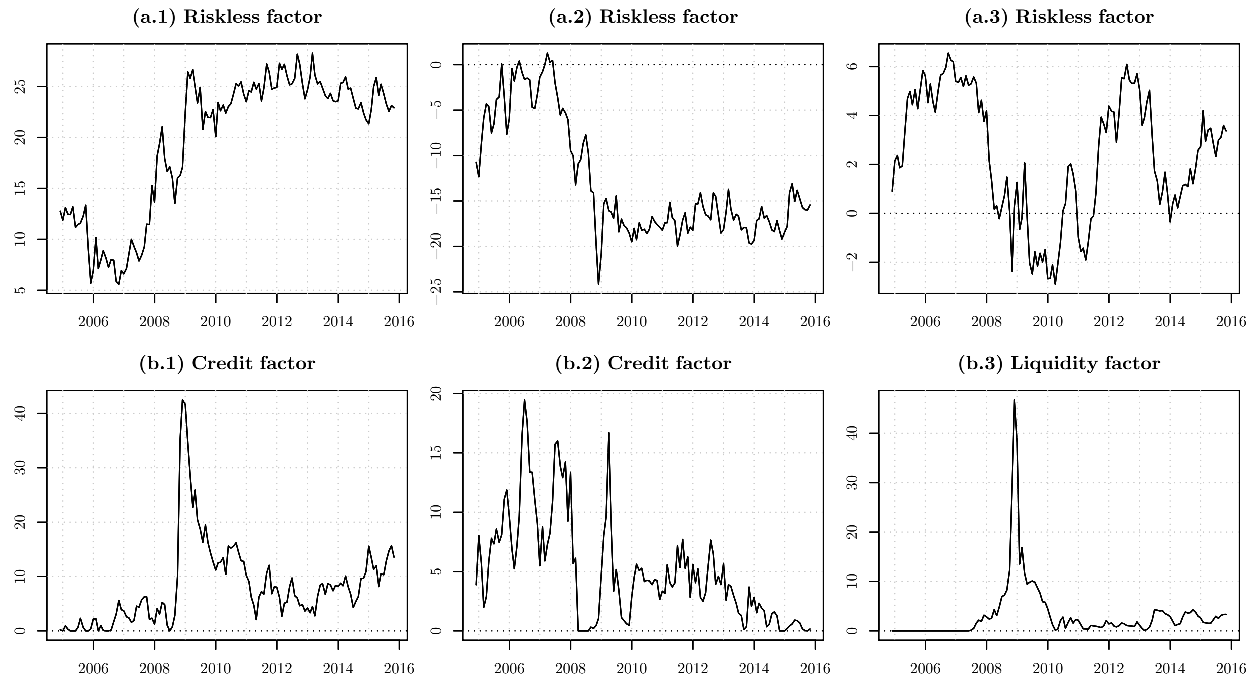


Figure IV: 6m OIS, inflation and liquidity proxy fitted values

The model is estimated with extended Kalman filter. Data range from 2004 to 2015. The black solid line presents the observation data used as input for estimation. The grey dashed line presents the fitted values produced through the filtered factors presented on Figure III.

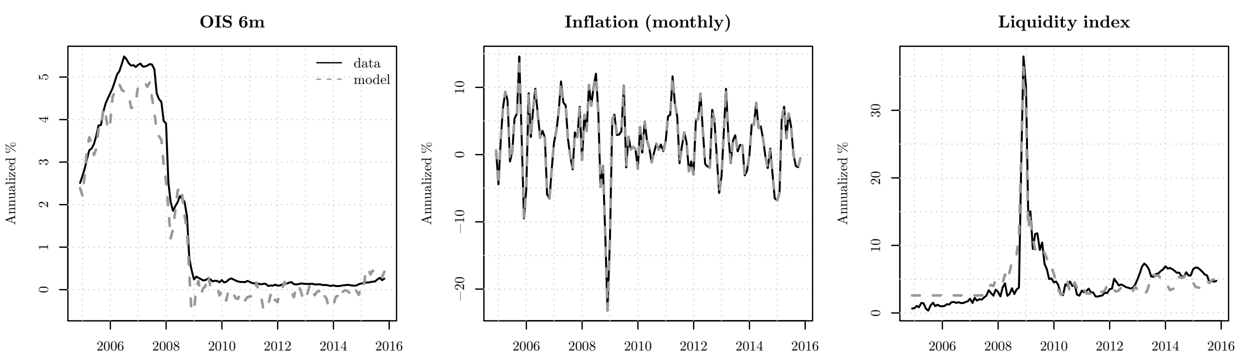


Figure V: Inflation-linked swaps fitted values

The model is estimated with extended Kalman filter. Data range from 2004 to 2015. The black solid line presents the observation data used as input for estimation. The grey dashed line presents the fitted values produced through the filtered factors presented on Figure III.

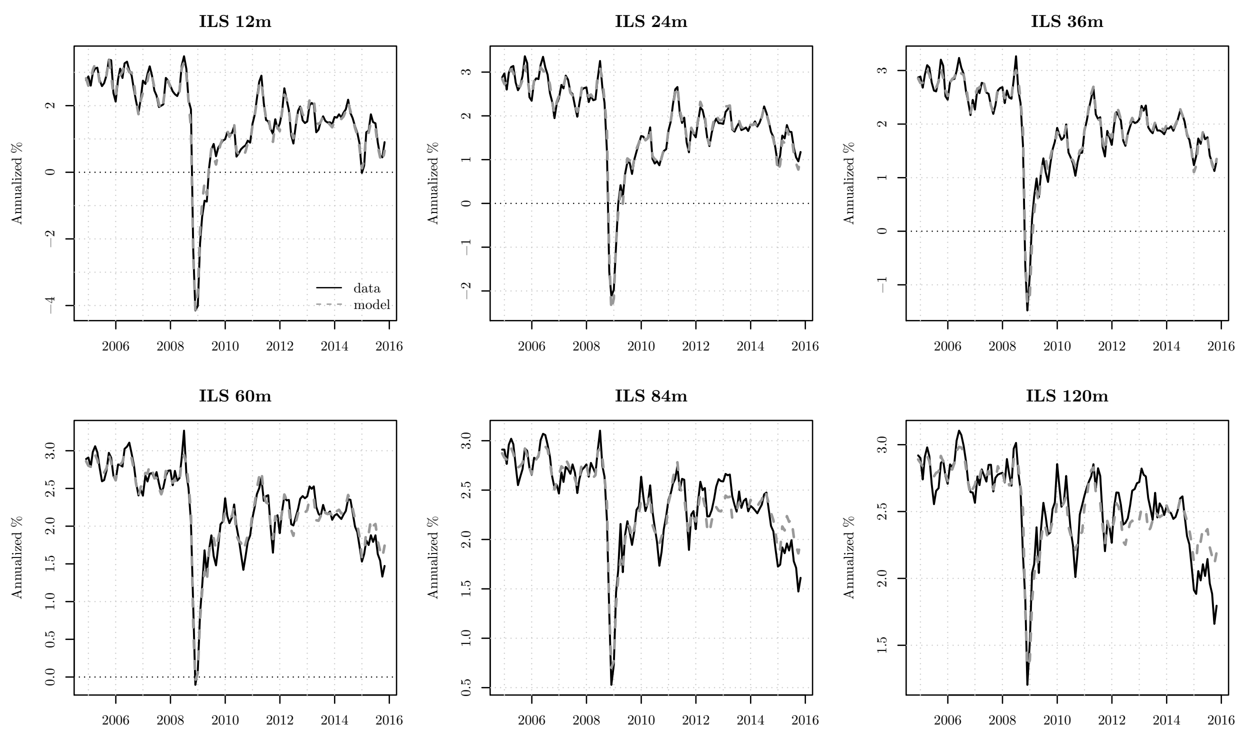


Figure VI: Nominal Treasuries fitted values

The model is estimated with extended Kalman filter. Data range from 2004 to 2015. The black solid line presents the observation data used as input for estimation. The grey dashed line presents the fitted values produced through the filtered factors presented on Figure III.

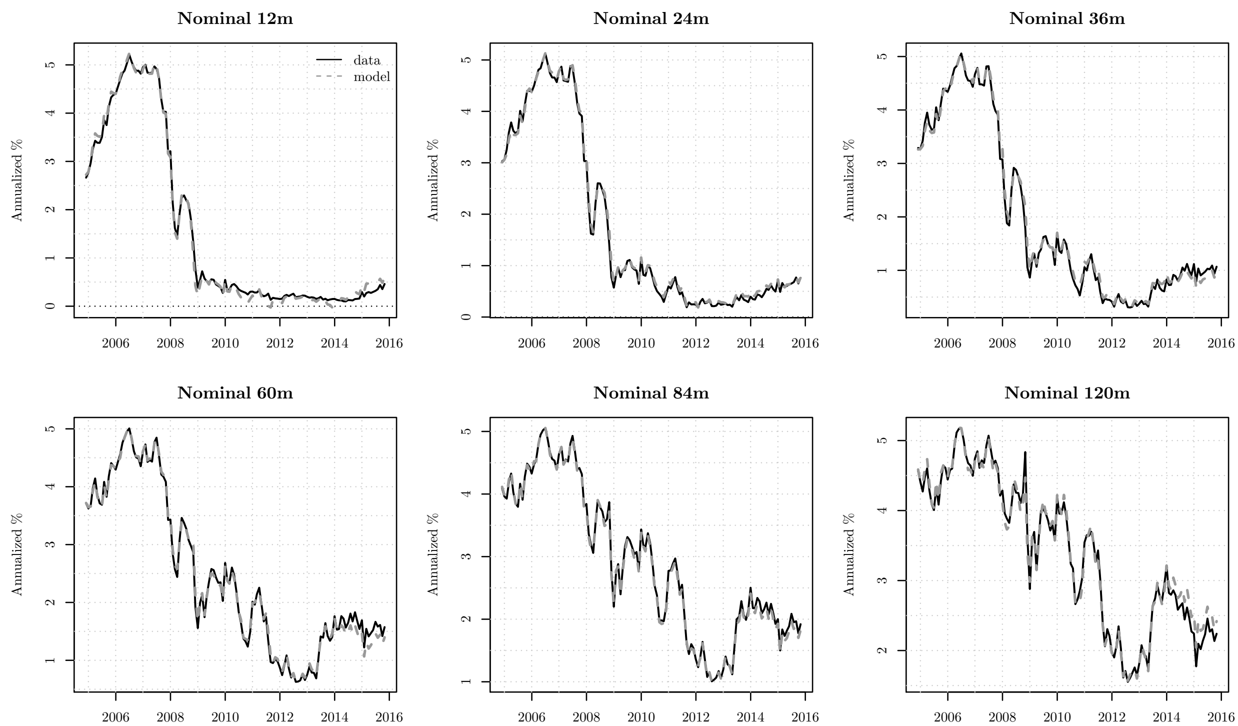


Figure VII: ILS-BEI spreads fitted values

The model is estimated with extended Kalman filter. Data range from 2004 to 2015. The black solid line presents the observation data used as input for estimation. The grey dashed line presents the fitted values produced through the filtered factors presented on Figure III.

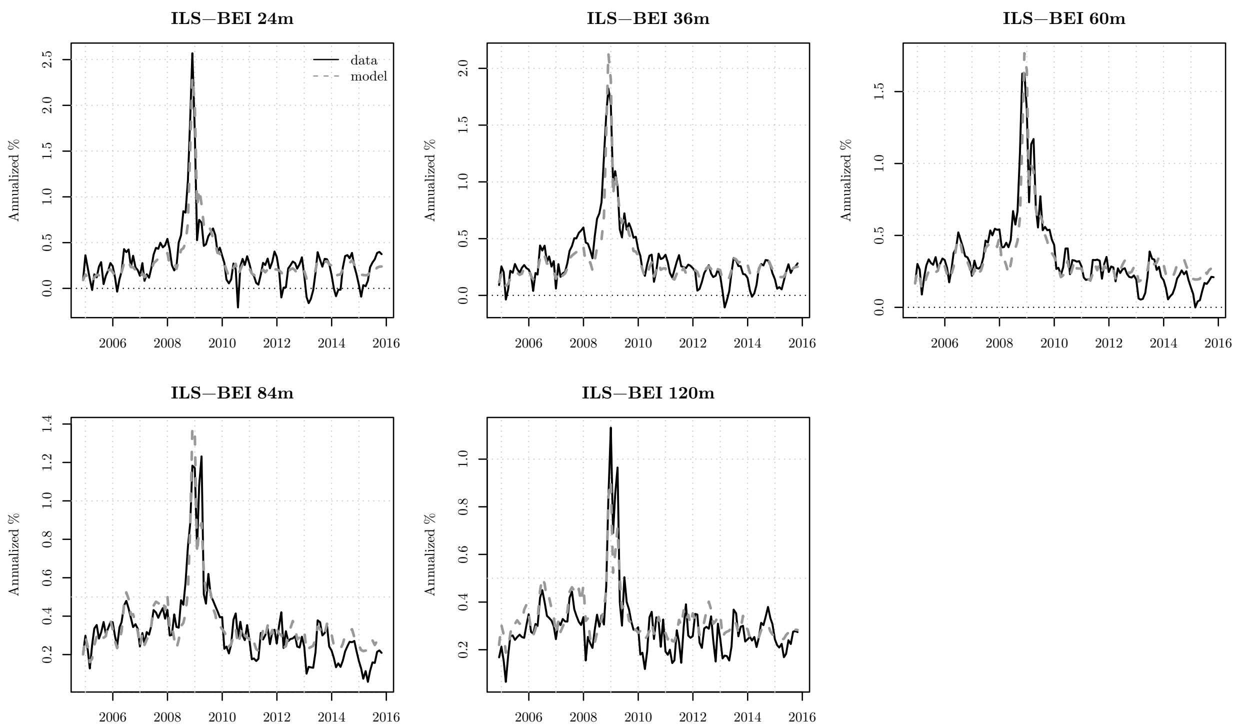


Figure VIII: U.S. sovereign CDS spreads fitted values

The model is estimated with extended Kalman filter. Data range from 2004 to 2015. The black solid line presents the observation data used as input for estimation. The grey dashed line presents the fitted values produced through the filtered factors presented on Figure III.

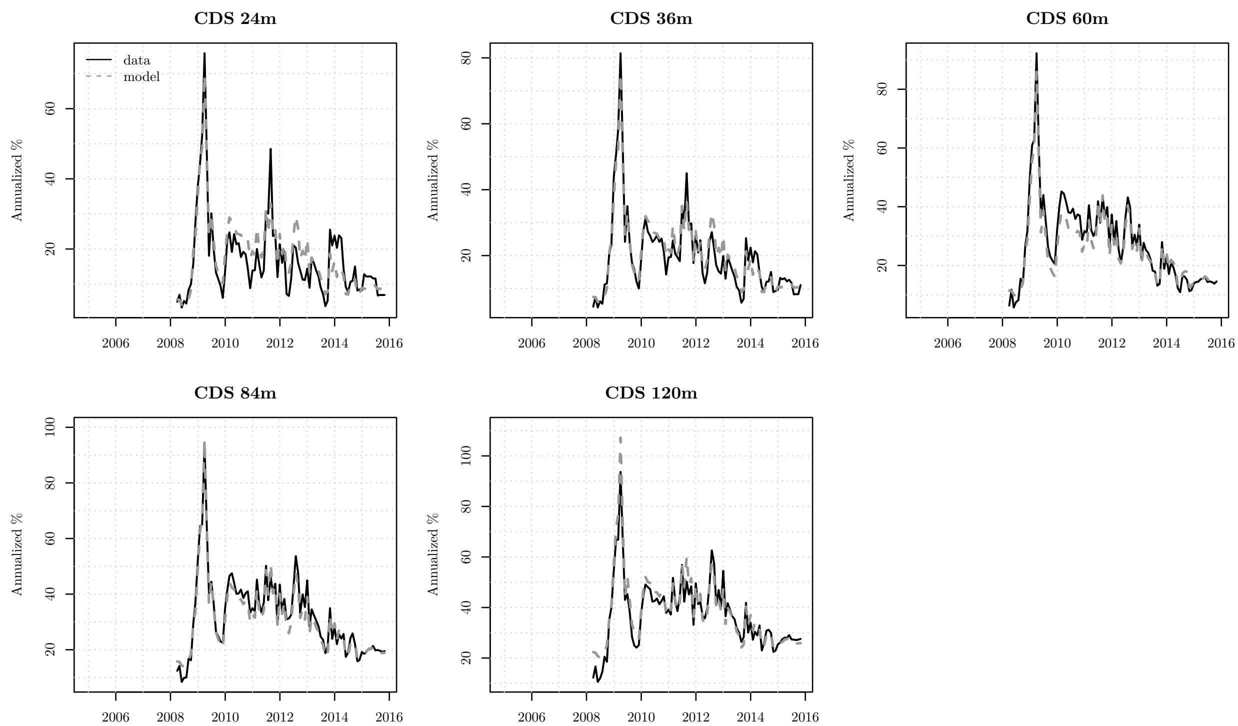


Figure IX: Credit component in ILS-BEI spreads

The model is estimated with extended Kalman filter. Data range from 2004 to 2015. The black solid line presents the observation data used as input for estimation. The grey dashed line presents the fitted values produced through the filtered factors presented on Figure III. The red component represents the ILS-BEI spreads that would be obtained would the liquidity intensity (or, equivalently, $y_t^{(\ell)}$ be 0 throughout the sample.

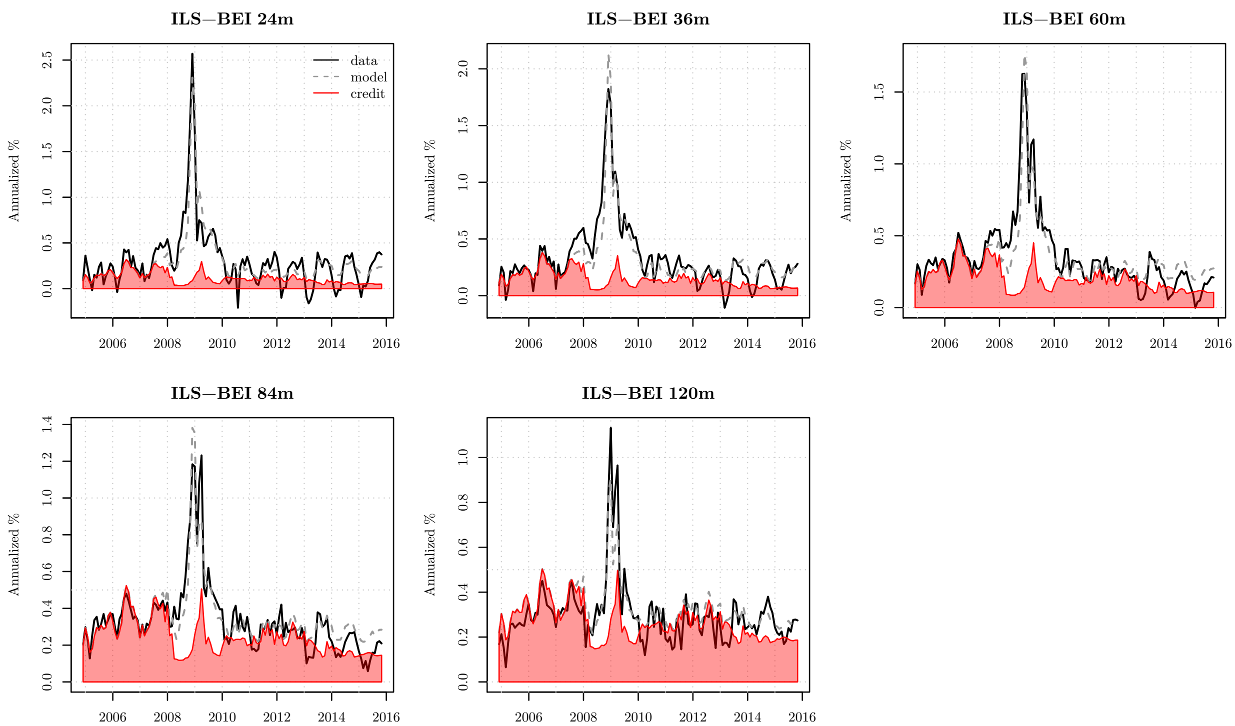


Figure X: Credit component in ILS-BEI spreads

The model is estimated with extended Kalman filter. Data range from 2004 to 2015. The red component represents the ILS-BEI spreads that would be obtained would the liquidity intensity (or, equivalently, $y_t^{(\ell)}$ be 0 throughout the sample, as a proportion of the fitted values presented on Figure VII as a grey dashed line.

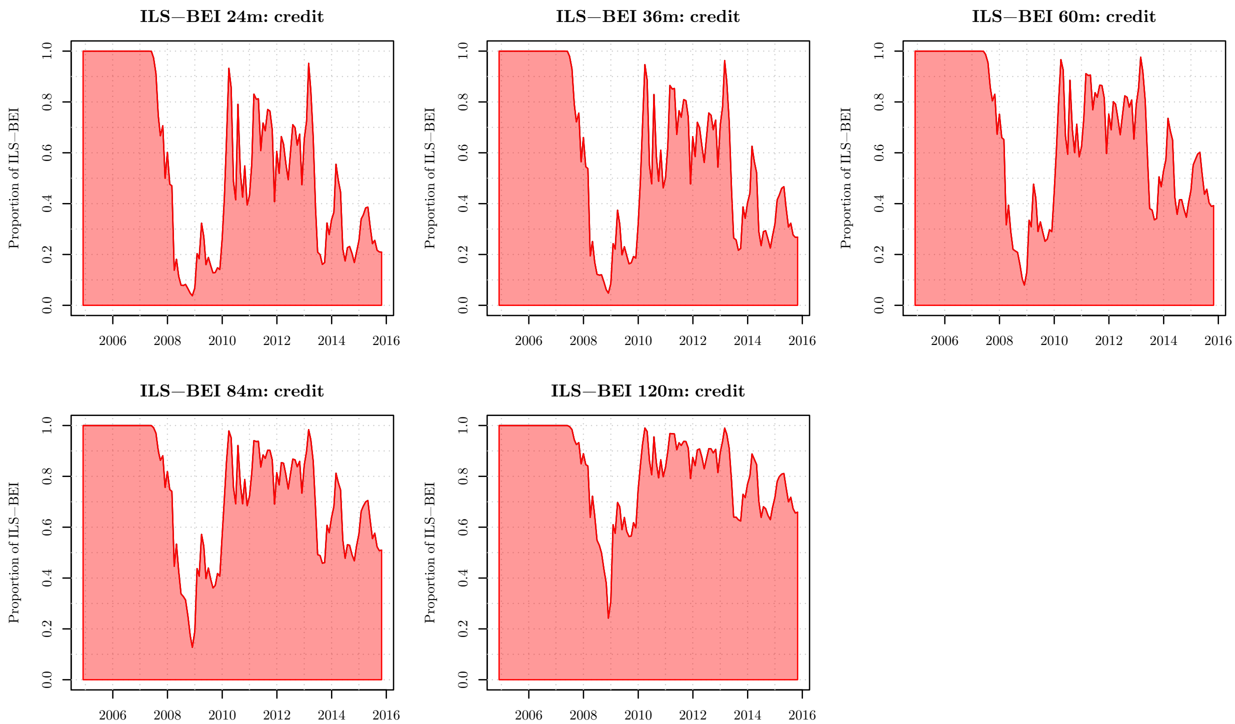


Figure XI: Total risk premia and credit risk premia in ILS-BEI spreads

The model is estimated with extended Kalman filter. Data range from 2004 to 2015. The green and blue components represent the total and credit risk premia, respectively, contained in ILS-BEI spreads. Total risk premia are obtained replacing the risk-neutral parameters by the physical ones and recomputing the ILS-BEI spreads given the estimated factors. The same procedure is applied for the credit premia, imposing the liquidity factor $y_t^{(\ell)}$ is equal to zero.

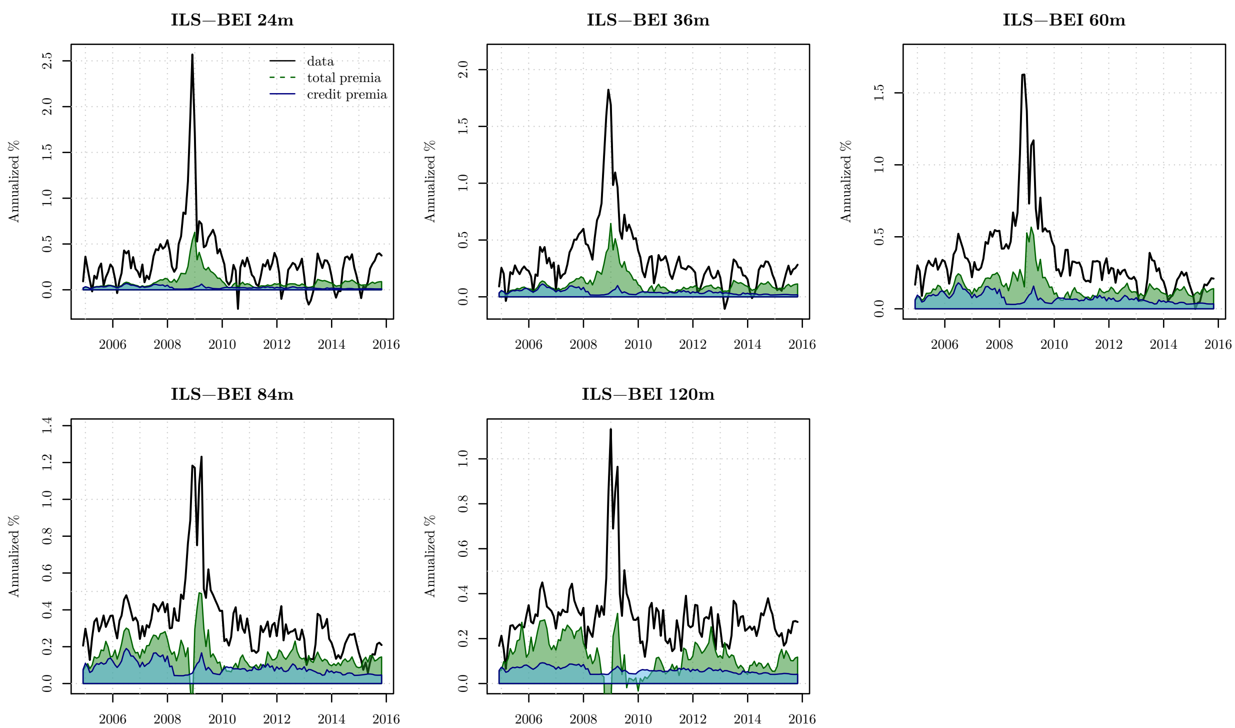
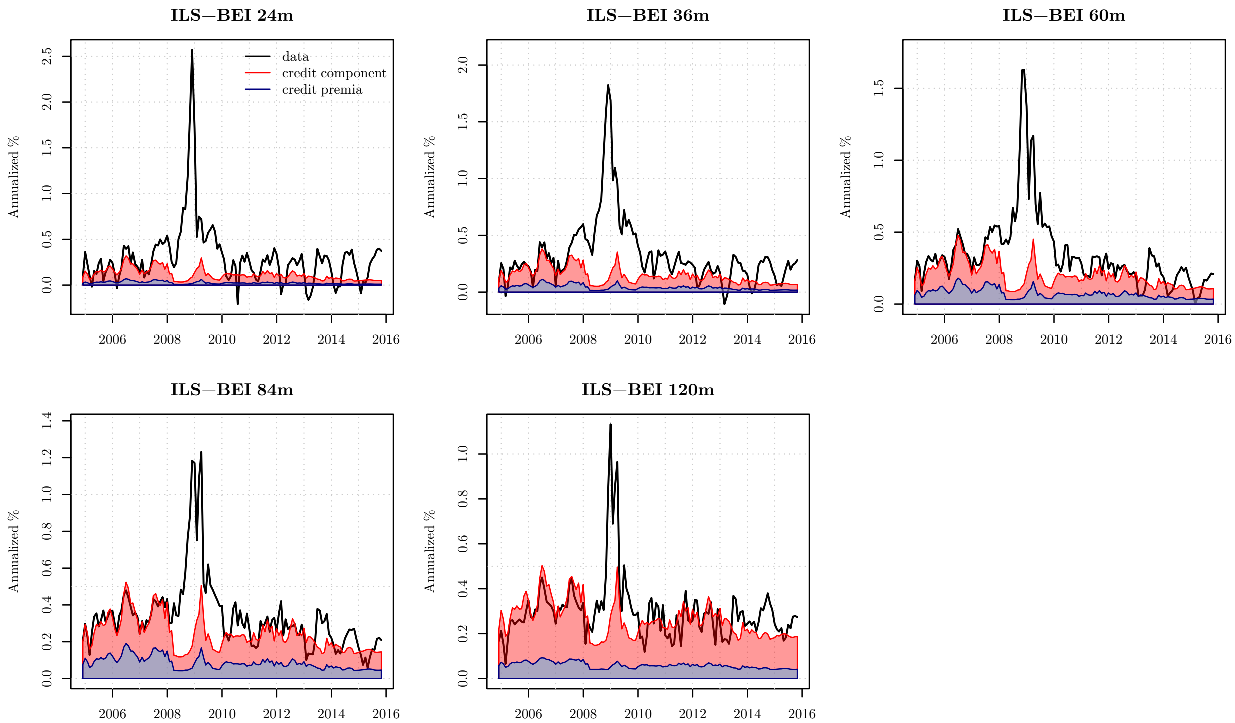


Figure XII: Credit component and credit risk premia in ILS-BEI spreads

The model is estimated with extended Kalman filter. Data range from 2004 to 2015. The red and blue components represent the credit component and risk premia, respectively, contained in ILS-BEI spreads. The credit component is computed by setting the liquidity factor $y_t^{(\ell)}$ to zero. Credit risk premia are obtained replacing the risk-neutral parameters by the physical ones and recomputing the ILS-BEI spreads given the estimated factors, imposing the liquidity factor $y_t^{(\ell)}$ is equal to zero.



A Appendix

A.1 Variable Descriptions

- HPW Noise, the measure of arbitrage capital availability proposed in Hu, Pan and Wang (2013). This measure is constructed as the root mean squared error in the observed yields of Treasury securities relative to those implied by a Nelson-Siegel-Svensson zero coupon curve across the term structure.[23] The measure takes into account the close relationship between availability of arbitrage capital and liquidity. Fleckenstein, Longstaff and Lustig (2014) posit that the inability of arbitrageurs to immediately eliminate arbitrage may have resulted in the divergence between nominal and inflation-protected securities markets. They suggest that this slow-moving capital hypothesis (Mitchell, Pedersen and Pulvino (2007) and Duffie (2010)) may allow arbitrage profits to persist. This HPW measure, which averages 3.52 basis points, rises to 20.47 basis points during the financial crisis.

- TIPS Noise, is the absolute average fitting error of the Nelson-Siegel-Svensson model estimated on the TIPS yield curve (see Gurkaynak, Sack and Wright (2010)). This variable mimics the HPW Noise measure for the TIPS market as opposed to nominal Treasuries. The series is obtained from the Board of Governors of the Federal Reserve and is computed at the daily frequency.[24] The series move between 0bps and 5 bps before the crisis, spike to 40 bps in late 2008 and average about 5 bps afterward. Since it represents the relative liquidity of TIPS, this measure is well-fitted to control for the slowly moving capital hypothesis of Fleckenstein, Longstaff and Lustig (2014).

- LIBOR-OIS, the spread between LIBOR and the overnight indexed swap rate. As shown in Table I, this spread, which averages 35 basis points over our sample, rose to 364 basis points during the crisis. This rise has been attributed to an increase in perceived counterparty credit risk in financial markets. Fleckenstein, Longstaff and Lustig (2014) suggest that their arbitrage profits could arise due to counterparty credit risk, especially if nominal Treasuries are viewed as safe haven assets. However, the authors suggest it is an unlikely explanation for their findings due to the collateralization of of swap contracts (Arora, Gandhi and Longstaff (2012)).

- OTR Difference, the yield difference between the 10-year off-the-run GSW par yield and the

[23] These data are obtained from Jun Pan's webpage, http://www.mit.edu/~junpan/

[24] We thank Richard Crump for providing us access to the data.

generic 10-year on-the-run yield from Bloomberg. During periods of stress, market participants may seek the most liquid securities, on-the-run government benchmark bonds, which accordingly often trade at a premium to an equivalent off-the-run bond.

- VIX, the CBOE volatility index. The VIX is often viewed as a measure of the market's perception of the quantity and/or price of risk in equity markets specifically, and financial markets as a whole. However, Nagel (2012) suggests that an increase in the VIX is associated with a higher premium for liquidity provision, and therefore a reduction in the amount of liquidity in the financial system. The VIX averages 22% over our sample period, with an increase to nearly 81% during the financial crisis.

A.2 Empirical Robustness

A.2.1 Pflueger and Viceira explanatory variables

In the approach of Pflueger and Viceira (2016), liquidity is proxied using three variables: the off-the-run spread (OTR), log relative volume in the TIPS and nominal Treasury markets (VOL), and the synthetic-cash spread, which is our variable ILS-BEI.In their main results, Pflueger and Viceira (2016) use the asset swap spread and use the ILS-BEI for robustness. Results using both variables are similar, and we use the ILS-BEI for simplicity and to complement our earlier results. The off-the-run spread is the difference between the 10 year off-the-run par yield and the 10-year on-the-run nominal yield from Bloomberg (USGG10YR). Relative volume in the two markets is measured using primary dealers' transaction volume from the New York Federal Reserve FR-2004 survey. Inflation expectations are measured using two variables, the median 10-year CPI forecast from the Survey of Professional Forecasters (CPI^e) and the Chicago Fed National Activity Index ($CFNAI$). The CPI forecast is available quarterly, and the $CFNAI$ is available monthly. We create a daily series using the most recently released data.Our results are similar in terms of signs and magnitude regardless of the data frequency; we also examine weekly and monthly data. However, statistical significance of some of the coefficients declines as we sample at coarser data frequencies.

A.2.2 Cheapest-to-Deliver

In the case of a credit event, the cheapest-to-deliver (CtD) obligation of the reference entity (here, the U.S. sovereign) will be a key determinant of the recovery rate on the underlying asset. In the auction process described in Section 2.2, all obligations deemed *deliverable* by the Determinations Committee are eligible to be sold. The protection buyer is incentivized to deliver the cheapest outstanding reference obligation and hence the final price at auction will largely be determined by the prevailing dealer quotes of this obligation. Therefore, CDS premia and the expected price of the CtD obligation would be expected to have a negative correlation. The CtD obligation may vary considerably over time and depend on the sovereign distance to default.

We examine the effects of CtD empirically in Table A1. We assume the general methodology of Klingler and Lando (2018) to identify the cheapest nominal Treasury on each day of our sample and include the following as a control variable:

$$CtD_t = 100 - \min(Price_t).$$

The regression result is essentially unchanged from our primary specification in Table II. In Column (1), CtD shows a weak, insignificant relationship with changes in ILS-BEI. The coefficient on CtD is negative and significant in Column (3) where we include all controls as well as a week and tenor fixed effect. Based on the construction of the CtD variable, a lower nominal bond price (higher yield) on any day is associated with a higher value of CtD. The results suggest higher Treasury yields are consistent with a narrower ILS-BEI, yet the coefficient on US CDS is essentially unchanged. There exists some ambiguity over what would be considered a deliverable obligation in the case of a sovereign credit event as this would ultimately be decided by the Determinations Committee. It is with minimal disagreement that nominal bonds up to a 30-year maturity would be accepted, yet the role of inflation-linked bonds, zero-coupon STRIPS, or sovereign guarantees (where U.S. sovereign gurantees the repayment of debt issued by another party) is less clear. Our empirical results are consistent whether we include or exclude TIPS in the definition of CtD.

A.2.3 Foreign Exchange Risk

Since the U.S. sovereign CDS contracts are denominated in euros, one could argue that there is foreign exchange risk embedded in CDS contracts which is also driving the variability in the changes in ILS-BEI. As noted by Chernov, Schmid and Schneider (2019), it makes sense for an investor looking for a protection against a U.S. default to obtain a payment in euros instead of dollars since it is likely that the dollar would greatly depreciate. The market for EUR-denominated CDSs is therefore more liquid than for USD-denominated CDSs. This gives rise to a so-called *quanto spread* that has been exploited to measure the depreciation risk upon default. To rule out euro-dollar exchange rate risk as an omitted variable in our baseline results, we perform panel regressions controlling for the exchange rate (risk) between the two currencies: the 5-year EURUSD basis swap spread ($EURUSD$) and the spot exchange rate ($Spot$) between the two currencies.

Table A2 summarizes the results. In columns (1) and (2), we regress changes in ILS-BEI on U.S. CDS spread, and $EURUSD$ or $Spot$, respectively. In Columns (3) and (4), we repeat the regressions after adding the same control variables as those used in Table II. Examining the coefficient loadings on U.S. CDS in the first row, we see that they are all positive and statistically significant across the board. The results presented in Table A2 of the Appendix are essentially unchanged and the coefficient on the CDS ranges from 0.18 to 0.24, as in our baseline specification. We conclude the exchange rate risk in euro CDS contracts is not driving our results.

A.2.4 Fleckenstein, Longstaff and Lustig (2014) Mispricing

Another measure of the relative pricing of nominal bonds vs. real bonds can be found in Fleckenstein, Longstaff and Lustig (2014). The authors replicate a nominal bond by matching the cash flows using a basket of inflation swaps, Treasury Strips, as well as a TIPS with similar maturity and coupon dates. In the absence of any market frictions, the price of the nominal bond and the price of the basket of replicating assets should be exactly the same. Surprisingly, this is not the case in the data, and we refer to the difference as Treasury-TIPS mispricing. Fleckenstein, Longstaff and Lustig (2014) document persistent mispricing between 2004 and 2009 in their sample that can be as high as \$20 per \$100 notional. We reproduce the matched bond pairs from their study and extend the sample period to 2015. We document that mispricing remains in the sample after the financial crisis, and it averages about \$3 per \$100 notional across bond pairs and across time.

We then perform our baseline panel regressions after replacing ILS-BEI with pairwise mispricing as the dependent variable. The results are shown in Table A3, which has a similar format as Table II with two exceptions. First, we add time-to-maturity (TTM) as a control variable in the panel. Second, instead of using a tenor fixed effect, column (7) employs a bond pair fixed effect. We also divide mispricing by 100 to convert mispricing from dollars to cents per \$1 notional. Similar to the first row of Table II, the estimated coefficients on U.S. CDS are positive and statistically significant under all specifications in Table A3. In Columns (6) and (7), with a full slate of control variables, a 1% increase in the CDS spread implies a 0.4 cent increase in the mispricing. This means the nominal bond trades approximately 40 cents rich compared to the basket of inflation swaps, Strips, and TIPS, per \$100 notional. This is also economically significant if you consider that the average mispricing is about \$3. Using the relative pricing of nominal and real bonds from Fleckenstein, Longstaff and Lustig (2014), we show that sovereign default risk embedded in CDS contracts is strongly correlated with the price differential between Treasury and TIPS. The direction of impact is also consistent with the effect on ILS-BEI: higher CDS spreads are associated with greater downward pressure on TIPS prices relative to Treasury prices.

A.2.5 Repo Premium

D'Amico and Pancost (2018) suggest the repo special spread for Treasury securities can aid in explaining numerous fixed income anomalies documented in the literature. Particularly during times of market stress, investors place a premium on the most liquid instruments resulting in a time-varying special collateral risk premium.[25] In Table A4 we test whether the repo premium plays a role in explaining the ILS-BEI spread. Based on data availability, the sample spans from January 2009 to October 2015. The coefficient on the repo premium is positive, but statistically insignificant across specifications. This is driven by the crisis era, which would be consistent with narrower Treasury yields, tigher BEI and hence a wider ILS-BEI spread. In untabulated results, we use the sample period beginning in 2010 and witness a negative, insignificant relationship between repo premiums and the ILS-BEI spread while coefficients on US CDS spreads are consistent with Table VI. It is possible that our use of zero-coupon smoothed yield curves to measure the ILS-BEI spread reduces the correlation with the repo premium that was derived from the cross-sections of outstanding Treasury securities.

A.2.6 TIPS Deflation Floors

One feature of TIPS that potentially can produce differential pricing relative to nominal Treasury bonds is the fact that it has a deflation floor. In our study, we show that the BEI spread narrows (ILS-BEI widens) when the CDS spread widens. To the extent that the U.S. CDS spread captures sovereign default risk of the U.S. government, this implies TIPS yields rise more than nominal yields when default risk is elevated. However, if it is the case that the option value of the deflation floor on TIPS is more valuable when default risk is high because deflation is more likely to happen during downturns, then the deflation floor on TIPS actually puts downward pressure on real yields in bad times. Therefore, the fact that we still see a narrowing of the BEI in the data when the CDS spread widens suggests that factors other than the deflation floor feature are driving the wedge between real and nominal bond prices.

[25] We thank Stefania D'Amico for this suggestion and Aaron Pancost for providing the data

A.3 Affine property and conditional moments of w_t

Let us compute the physical conditional moment-generating function of w_t applied in $u = \left(u_x', u_y', u_\delta'\right)'$.

$$\begin{aligned}\varphi_{w_t}^{\mathbb{P}}(u) &:= \mathbb{E}_t^{\mathbb{P}}\left[\exp\left(u'w_{t+1}\right)\right]\\ &= \exp\left\{u_x'\left(\mu+\Phi x_t\right)+\frac{1}{2}u_x'\Sigma u_x\right\}\mathbb{E}_t^{\mathbb{P}}\left\{\mathbb{E}_t^{\mathbb{P}}\left[\exp\left(u_y'y_{t+1}+u_\delta'\delta_{t+1}\right)\left|y_{t+1}\right.\right]\right\}\\ &= \exp\left\{u_x'\left(\mu+\Phi x_t\right)+\frac{1}{2}u_x'\Sigma u_x\right\}\mathbb{E}_t^{\mathbb{P}}\left[\exp\left(\left(\beta_\lambda\frac{\operatorname{diag}(\mathsf{c}_\delta)u_\delta}{\mathbf{1}-\operatorname{diag}(\mathsf{c}_\delta)u_\delta}+u_y\right)'y_{t+1}\right)\right]\end{aligned}$$

where the fraction is an abuse of notation for an element by element ratio and:

$$\beta_\lambda = \begin{pmatrix}\beta_\lambda^{(c)} & \mathbf{0}\\ \mathbf{0} & \beta_\lambda^{(\ell)}\end{pmatrix}.$$

Thus, denoting by $\widetilde{u}_y = \left(\beta_\lambda\frac{\operatorname{diag}(\mathsf{c}_\delta)u_\delta}{\mathbf{1}-\operatorname{diag}(\mathsf{c}_\delta)u_\delta}+u_y\right)$, we have:

$$\varphi_{w_t}^{\mathbb{P}}(u) = \exp\left\{u_x'\left(\mu+\Phi x_t\right)+\frac{1}{2}u_x'\Sigma u_x+\left(\frac{\operatorname{diag}(\mathsf{c}_y)\widetilde{u}_y}{\mathbf{1}-\operatorname{diag}(\mathsf{c}_y)\widetilde{u}_y}\right)'\beta_y\,y_t-\nu'\log\left[\mathbf{1}-\operatorname{diag}(\mathsf{c}_y)\widetilde{u}_y\right]\right\}, \tag{27}$$

which is an exponential-affine function of x_t and y_t, thus of w_t by extension. The conditional mean of w_t is then given by:

$$\begin{aligned}\mathbb{E}_t^{\mathbb{P}}\left(x_{t+1}\right) &= \mu+\Phi\,x_t\\ \mathbb{E}_t^{\mathbb{P}}\left(y_{t+1}\right) &= \operatorname{diag}(\mathsf{c}_y)\left(\nu+\beta_y\,y_t\right)\\ \mathbb{E}_t^{\mathbb{P}}\left(\delta_{t+1}\right) &= \mathbb{E}_t^{\mathbb{P}}\left[\mathbb{E}_t^{\mathbb{P}}\left(\delta_{t+1}\left|y_{t+1}\right.\right)\right] = \operatorname{diag}(\mathsf{c}_\delta)\beta_\lambda'\mathbb{E}_t^{\mathbb{P}}\left(y_{t+1}\right).\end{aligned}$$

For notational convenience, we introduce the block matrix Q of size $N\times N$ defined as:

$$\begin{pmatrix} I_{N_x} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \operatorname{diag}\left(\mathsf{c}_y\right) & \mathbf{0}\\ \mathbf{0} & \operatorname{diag}\left(\mathsf{c}_\delta\right)\beta_\lambda\operatorname{diag}\left(\mathsf{c}_y\right) & \operatorname{diag}\left(\mathsf{c}_\delta\right)\end{pmatrix}$$

We obtain that

$$\mathbb{E}_t^{\mathbb{P}}(w_{t+1}) = \Psi_0+\Psi\,w_t\,,$$

where:

$$\Psi_0 = Q \times \begin{pmatrix} \mu \\ \nu \\ \mathbf{0} \end{pmatrix} \quad \text{and} \quad \Psi = Q \times \begin{pmatrix} \Phi & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \beta_y & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} . \tag{28}$$

Let us now turn to the conditional variance. x_t is independent from y_t and δ_t, and its conditional covariance matrix is given by Σ. Then, using the properties of gamma variables, we have:

$$\mathbb{V}_t^{\mathbb{P}}(y_{t+1}) = \text{diag}(\mathsf{c}_y)^2 \times \text{diag}(\nu + 2\beta_y \, y_t) \; .$$

Using the law of total variance, we can express the conditional variance of δ_t as:

$$\begin{aligned} \mathbb{V}_t^{\mathbb{P}}(\delta_{t+1}) &= \mathbb{V}_t^{\mathbb{P}}\left[\mathbb{E}_t^{\mathbb{P}}\left(\delta_{t+1} \middle| y_{t+1}\right)\right] + \mathbb{E}_t^{\mathbb{P}}\left[\mathbb{V}_t^{\mathbb{P}}\left(\delta_{t+1} \middle| y_{t+1}\right)\right] \\ &= \mathbb{V}_t^{\mathbb{P}}\left[\text{diag}(\mathsf{c}_\delta)\,\beta_\lambda' \, y_{t+1}\right] + \mathbb{E}_t^{\mathbb{P}}\left[2\text{diag}(\mathsf{c}_\delta)^2 \, \text{diag}\left(\beta_\lambda' \, y_{t+1}\right)\right] \\ &= \text{diag}(\mathsf{c}_\delta)\,\beta_\lambda' \, \mathbb{V}_t^{\mathbb{P}}(y_{t+1})\,\beta_\lambda \text{diag}(\mathsf{c}_\delta) + 2\text{diag}(\mathsf{c}_\delta)^2 \, \text{diag}\left(\beta_\lambda' \, \mathbb{E}_t^{\mathbb{P}}\left[y_{t+1}\right]\right) \\ &= \text{diag}(\mathsf{c}_\delta)\,\beta_\lambda' \, \mathbb{V}_t^{\mathbb{P}}(y_{t+1})\,\beta_\lambda \text{diag}(\mathsf{c}_\delta) + 2\text{diag}(\mathsf{c}_\delta)^2 \, \text{diag}\left[\beta_\lambda' \, \text{diag}(\mathsf{c}_y)\,(\nu + \beta_y \, y_t)\right] \; . \end{aligned}$$

Last, the conditional covariance between y_t and δ_t is given by:

$$\begin{aligned} \mathbb{C}ov_t^{\mathbb{P}}(y_{t+1}, \, \delta_{t+1}) &= \mathbb{C}ov_t^{\mathbb{P}}\left(y_{t+1}, \, \mathbb{E}_t^{\mathbb{P}}\left[\delta_{t+1} \middle| y_{t+1}\right]\right) + \mathbb{E}_t^{\mathbb{P}}\left[\mathbb{C}ov_t^{\mathbb{P}}\left(y_{t+1}, \, \delta_{t+1} \middle| y_{t+1}\right)\right] \\ &= \mathbb{C}ov_t^{\mathbb{P}}\left(y_{t+1}, \, \text{diag}(\mathsf{c}_\delta)\,\beta_\lambda' \, y_{t+1}\right) \\ &= \mathbb{V}_t^{\mathbb{P}}(y_{t+1})\,\beta_\lambda \, \text{diag}(\mathsf{c}_\delta) \; . \end{aligned}$$

Putting all results together, we obtain:

$$\Omega_{t-1} = \mathbb{V}_t^{\mathbb{P}}(w_{t+1}) = Q \times \begin{pmatrix} \Sigma & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \text{diag}(\nu + 2\beta_y \, y_t) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & 2\text{diag}\left[\beta_\lambda' \, \text{diag}(\mathsf{c}_y)\,(\nu + \beta_y \, y_t)\right] \end{pmatrix} \times Q' \, . \tag{29}$$

We obtain unconditional moments by assuming stationarity of w_t:

$$\begin{aligned} \mathbb{E}^{\mathbb{P}}(w_t) &= (I_N - \Psi)^{-1} \, \Psi_0 \\ \text{Vec}\left[\mathbb{V}^{\mathbb{P}}(w_t)\right] &= \left[I_{N^2} - (Q \otimes Q)\,(\Psi \otimes \Psi)\right]^{-1} \times \left[\Omega_0 + \Omega \mathbb{E}^{\mathbb{P}}(y_t)\right] \, , \end{aligned}$$

where Ω_0 and Ω are such that:

$$\text{Vec}\begin{pmatrix} \Sigma & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \text{diag}\,(\nu + 2\beta_y\, y_t) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & 2\text{diag}\,[\beta_\lambda'\,\text{diag}\,(\mathsf{c}_y)\,(\nu + \beta_y\, y_t)] \end{pmatrix} = \Omega_0 + \Omega y_t\,.$$

A.4 Affine risk-neutral property

To obtain the affine property under $\mathbb{Q}$ we need to proceed to the change of measure implied by the SDF specification of Equation (12). Since x_t is independent from y_t and δ_t and that the SDF does not incorporate cross-terms, we can proceed to its change of measure separately from that of $(y_t,\, \delta_t)$.

Our specification of x_t dynamics and the SDF that depends on x_t is that of a standard Gaussian affine term structure model with time varying prices of risk. We can thus directly apply the standard result that:

$$x_t = \mu^{\mathbb{Q}} + \Phi^{\mathbb{Q}} x_{t-1} + \sqrt{\Sigma}\,\varepsilon_t^{\mathbb{Q}}\,, \quad \text{where} \quad \varepsilon_t^{\mathbb{Q}} \sim \mathcal{N}\left(0, I_{N_x}\right)\,, \tag{30}$$

and the risk-neutral parameters are given by:

$$\mu^{\mathbb{Q}} = \mu + \Sigma\,\theta_{0,x}, \quad \Phi^{\mathbb{Q}} = \Phi + \Sigma\,\Theta_x\,. \tag{31}$$

For the change of measure associated with the default and liquidity risk variables, we rely on Propositions 2.5-2.6 of Monfort et al. (2020), and we have that the risk-neutral intensities are equal to the physical intensities (there is no pricing of "surprise", i.e. the SDF does not depend on δ_t), and the risk-neutral dynamics of y_t is given by a vector autoregressive gamma process, such that:

$$y_t\,|\,y_{t-1} \overset{\mathbb{Q}}{\sim} \Gamma_\nu\left(\beta_y^{\mathbb{Q}}\, y_{t-1},\, \mathsf{c}_y^{\mathbb{Q}}\right)\,, \quad \text{where} \quad \beta_y^{\mathbb{Q}} = \beta_y\,\text{diag}\left(\frac{\mathbf{1}}{\mathbf{1} - \text{diag}(\theta_y)\,\mathsf{c}_y}\right), \quad \mathsf{c}_y^{\mathbb{Q}} = \frac{\mathsf{c}_y}{\mathbf{1} - \text{diag}(\theta_y)\,\mathsf{c}_y}\,. \tag{32}$$

Hence, since the classes of distributions are the same under the risk-neutral measure, w_t is an affine process under the risk-neutral measure and its conditional moment generating function is given by:

$$\begin{aligned} \varphi_{w_t}^{\mathbb{Q}}(u) &:= \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(u' w_{t+1}\right)\right] \\ &= \exp\left\{u_x'\left(\mu^{\mathbb{Q}} + \Phi^{\mathbb{Q}} x_t\right) + \frac{1}{2}u_x'\Sigma u_x + \left(\frac{\text{diag}(\mathsf{c}_y^{\mathbb{Q}})\widetilde{u}_y^{\mathbb{Q}}}{\mathbf{1} - \text{diag}(\mathsf{c}_y^{\mathbb{Q}})\widetilde{u}_y^{\mathbb{Q}}}\right)' \beta_y^{\mathbb{Q}}\, y_t - \nu' \log\left[\mathbf{1} - \text{diag}(\mathsf{c}_y^{\mathbb{Q}})\widetilde{u}_y^{\mathbb{Q}}\right]\right\}, \end{aligned}$$

where

$$\widetilde{u}_y^{\mathbb{Q}} = \widetilde{u}_y = \left(\beta_\lambda \frac{\mathrm{diag}(\mathsf{c}_\delta) u_\delta}{\mathbf{1} - \mathrm{diag}(\mathsf{c}_\delta) u_\delta} + u_y \right) .$$

Building on the property of affine processes, we have that the multi-horizon moment generating function of w_t is also an exponential-affine function of w_t under the risk-neutral measure. Let us introduce the following notation:

$$\varphi_{w_t}^{\mathbb{Q}}(u) = \exp\left(A^{\mathbb{Q}}(u) + B^{\mathbb{Q}}(u)' \, w_t \right) .$$

We have that:

$$\varphi_{w_t}^{\mathbb{Q}}\left(u_1, \ldots, u_n\right) \;\; := \;\; \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(\sum_{i=j}^{n} u_j' \, w_t \right) \right] = \exp\left[\mathcal{A}_n^{\mathbb{Q}}\left(u_1, \ldots, u_n\right) + \mathcal{B}_n^{\mathbb{Q}}\left(u_1, \ldots, u_n\right)' \, w_t \right] ,$$

where $\mathcal{A}_0^{\mathbb{Q}}\left(u_1, \ldots, u_n\right) = \mathbf{0}$ and $\mathcal{B}_n^{\mathbb{Q}}\left(u_1, \ldots, u_n\right) = \mathbf{0}$ and the loadings are defined through the following recursions:

$$\begin{aligned} \mathcal{A}_n^{\mathbb{Q}}\left(u_1, \ldots, u_n\right) &= A^{\mathbb{Q}}\left(u_1 + \mathcal{B}_{n-1}^{\mathbb{Q}}\left(u_2, \ldots, u_n\right) \right) + \mathcal{A}_{n-1}^{\mathbb{Q}}\left(u_2, \ldots, u_n\right) \\ \mathcal{B}_n^{\mathbb{Q}}\left(u_1, \ldots, u_n\right) &= B^{\mathbb{Q}}\left(u_1 + \mathcal{B}_{n-1}^{\mathbb{Q}}\left(u_2, \ldots, u_n\right) \right) . \end{aligned}$$

Equation (16) defines this multi-horizon moment generating function when all $n-1$ first arguments are equal, i.e. $u_1 = u_2 = \ldots = u_{n-1} = u$ and $u_n = v$. Thus, our notation $\mathcal{A}_n^{\mathbb{Q}}(u, v)$ and $\mathcal{B}_n^{\mathbb{Q}}(u, v)$ can be obtained through the above recursions by calculating $\mathcal{A}_n^{\mathbb{Q}}(u, \ldots, u, v)$ and $\mathcal{B}_n^{\mathbb{Q}}(u, \ldots, u, v)$.

A.5 Pricing formulas for riskless nominal and real bonds

The price of riskless nominal bonds and TIPS is given by:

$$\begin{aligned} D_t^{(n)} &= \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\sum_{j=0}^{n-1} r_{t+j} \right) \right] = e^{-n\kappa_0^{(r)}} \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\sum_{j=0}^{n-1} \kappa^{(r)\prime} \, w_{t+j} \right) \right] \\ D_t^{*(n)} &= \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\sum_{j=0}^{n-1} r_{t+j} - \pi_{t+j+1} \right) \right] \\ &= e^{-n\left(\kappa_0^{(r)} - \kappa_0^{(\pi)} - \kappa^{(r)\prime} \, w_t\right)} \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\sum_{j=1}^{n-1} \left(\kappa^{(r)} - \kappa^{(\pi)} \right)' \, w_{t+j} + \kappa^{(\pi)\prime} \, w_{t+n} \right) \right] . \end{aligned}$$

Thus, using our notation for the multi-horizon moment generating function of w_t under the risk-neutral measure, these expectations can be transformed as:

$$
\begin{aligned}
D_t^{(n)} &= \exp\left\{-n\kappa_0^{(r)} + \mathcal{A}_n^{\mathbb{Q}}\left(-\kappa^{(r)}, \mathbf{0}\right) + \left[\mathcal{B}_n^{\mathbb{Q}}\left(-\kappa^{(r)}, \mathbf{0}\right) - \kappa^{(r)}\right]' w_t\right\} \\
D_t^{*(n)} &= \exp\left\{-n\left(\kappa_0^{(r)} - \kappa_0^{(\pi)}\right) + \mathcal{A}_n^{\mathbb{Q}}\left(\kappa^{(\pi)} - \kappa^{(r)}, \kappa^{(\pi)}\right) + \left[\mathcal{B}_n^{\mathbb{Q}}\left(\kappa^{(\pi)} - \kappa^{(r)}, \kappa^{(\pi)}\right) - \kappa^{(r)}\right]' w_t\right\}.
\end{aligned}
$$

A.6 Pricing formulas for nominal treasuries and TIPS

Let us first focus on nominal bonds. We rewrite Equation (20) for convenience:

$$
\begin{aligned}
B_t^{(n)} &= \sum_{i=1}^{n} \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\sum_{j=0}^{i-1} r_{t+j}\right)\mathcal{P}_{t+i}^{(n_r)} \times \left(\mathbb{1}\left\{\sum_{j=0}^{i-1}\delta_{t+j}^{(c)} = 0\right\} - \mathbb{1}\left\{\sum_{j=0}^{i}\delta_{t+j}^{(c)} = 0\right\}\right)\right] \\
&+ \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\sum_{j=0}^{n-1} r_{t+j}\right)\mathbb{1}\left\{\sum_{j=0}^{n}\delta_{t+j}^{(c)} = 0\right\}\right].
\end{aligned}
$$

We start from the premise that the recovery payment is an exponential-affine function of w_t given by $\exp\left(\mathsf{A}_{n_r} + \mathsf{B}_{n_r}' w_t\right)$. Focusing on the first indicator term in the above equation, we can write:

$$
\begin{aligned}
& \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\sum_{j=0}^{i-1} r_{t+j}\right)\mathcal{P}_{t+i}^{(n_r)} \times \mathbb{1}\left\{\sum_{j=0}^{i-1}\delta_{t+j}^{(c)} = 0\right\}\right] \\
= {} & \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(\mathsf{A}_{n_r} + \mathsf{B}_{n_r}' w_{t+i} - i\kappa_0^{(r)} - \sum_{j=0}^{i-1}\kappa^{(r)\prime} w_{t+j}\right) \times \mathbb{1}\left\{\sum_{j=0}^{i-1}\delta_{t+j}^{(c)} = 0\right\}\right].
\end{aligned}
$$

Using the lemma 3.1 of Monfort et al. (2020), we have:

$$
\begin{aligned}
& \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(\mathsf{A}_{n_r} + \mathsf{B}_{n_r}' w_{t+i} - i\kappa_0^{(r)} - \sum_{j=0}^{i-1}\kappa^{(r)\prime} w_{t+j}\right) \times \mathbb{1}\left\{\sum_{j=0}^{i-1}\delta_{t+j}^{(c)} = 0\right\}\right] \\
= {} & \lim_{\mathsf{u}\to+\infty} \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(\mathsf{A}_{n_r} + \mathsf{B}_{n_r}' w_{t+i} - i\kappa_0^{(r)} - \sum_{j=0}^{i-1}\left(\kappa^{(r)\prime} w_{t+j} + \mathsf{u}\delta_{t+j}^{(c)}\right)\right)\right] \\
= {} & \lim_{\mathsf{u}\to+\infty} \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(\mathsf{A}_{n_r} + \mathsf{B}_{n_r}' w_{t+i} - i\kappa_0^{(r)} - \sum_{j=0}^{i-1}\left(\kappa^{(r)} + \mathsf{u}\mathbf{e}_c\right)' w_{t+j}\right)\right] \\
= {} & \lim_{\mathsf{u}\to+\infty} \exp\left\{\mathsf{A}_{n_r} - i\kappa_0^{(r)} + \mathcal{A}^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c, \mathsf{B}_{n_r}\right) + \left[\mathcal{B}^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c, \mathsf{B}_{n_r}\right) - \kappa^{(r)} - \mathsf{u}\mathbf{e}_c\right]' w_t\right\}
\end{aligned}
$$

Applying the same logic to the remaining terms, and assuming default has not occurred at date t, we obtain the result of Equation (22):

$$\begin{aligned} B_t^{(n)} &= \lim_{\mathsf{u}\to+\infty} e^{\mathsf{A}_{n_r}-\kappa^{(r)\prime} w_t} \sum_{i=1}^{n} e^{-i\kappa_0^{(r)}} \left[\exp\left\{\mathcal{A}_i^{\mathbb{Q}}\left(-\kappa^{(r)}-\mathsf{u}\mathbf{e}_c,\,\mathsf{B}_{n_r}\right)+\mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)}-\mathsf{u}\mathbf{e}_c,\,\mathsf{B}_{n_r}\right)' w_t\right\}\right. \\ &- \left.\exp\left\{\mathcal{A}_i^{\mathbb{Q}}\left(-\kappa^{(r)}-\mathsf{u}\mathbf{e}_c,\,\mathsf{B}_{n_r}-\mathsf{u}\mathbf{e}_c\right)+\mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)}-\mathsf{u}\mathbf{e}_c,\,\mathsf{B}_{n_r}-\mathsf{u}\mathbf{e}_c\right)' w_t\right\}\right] \\ &+ \exp\left\{-n\kappa_0^{(r)}+\mathcal{A}_n^{\mathbb{Q}}\left(-\kappa^{(r)}-\mathsf{u}\mathbf{e}_c,\,-\mathsf{u}\mathbf{e}_c\right)+\left[\mathcal{B}_n^{\mathbb{Q}}\left(-\kappa^{(r)}-\mathsf{u}\mathbf{e}_c,\,-\mathsf{u}\mathbf{e}_c\right)-\kappa^{(r)}\right]' w_t\right\}, \end{aligned}$$

Let us now turn to TIPS valuation. Again, for convenience, we rewrite the general pricing formula given by Equation (21) below:

$$\begin{aligned} B_t^{*(n)} &= \sum_{i=1}^{n} \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\sum_{j=0}^{i-1} r_{t+j}-\rho^*\pi_{t+j+1}\right)\mathcal{P}_{t+i}^{(n_r)} \times \left(\mathbb{1}\left\{\sum_{j=0}^{i-1}\delta_{t+j}^{(c)}=0\right\}-\mathbb{1}\left\{\sum_{j=0}^{i}\delta_{t+j}^{(c)}=0\right\}\right)\right] \\ &+ \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\sum_{j=0}^{i-1} r_{t+j}-\pi_{t+j+1}\right)e^{-\delta_{t+i}^{(\ell)}} \times \mathbb{1}\left\{\sum_{j=0}^{i}\delta_{t+j}^{(c)}=0\right\}\left(\mathbb{1}\left\{\sum_{j=0}^{i-1}\delta_{t+j}^{(\ell)}=0\right\}-\mathbb{1}\left\{\sum_{j=0}^{i}\delta_{t+j}^{(\ell)}=0\right\}\right)\right] \\ &+ \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\sum_{j=0}^{n-1} r_{t+j}-\pi_{t+j+1}\right)\mathbb{1}\left\{\sum_{j=0}^{n}\delta_{t+j}^{(c)}=0\right\}\mathbb{1}\left\{\sum_{j=0}^{n}\delta_{t+j}^{(\ell)}=0\right\}\right]. \end{aligned}$$

Let us first justify these different terms. The first row corresponds to the expected discounted ($e^{-r_t-\ldots-r_{t+i-1}}$) recovery cashflow ($e^{\rho^*(\pi_{t+1}+\ldots+\pi_{t+i})}\mathcal{P}_{t+i}^{(n_r)}$) in case default happens at date $t+i$ exactly (difference of indicators). The second row is the expected discounted ($e^{-r_t-\ldots-r_{t+i-1}}$) liquidity recovery cashflow (recovery rate $e^{-\delta_{t+i}^{(\ell)}}$ on face value $e^{\pi_{t+1}+\ldots+\pi_{t+i}}$) if no default has happened and liquidity event happens exactly at $t+i$. The last row is the expected discounted inflated face value if no credit or liquidity events have happened. In terms of asset pricing, assuming no default at date t, the first term of this equation can be detailed as:

$$\begin{aligned}
&\mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\sum_{j=0}^{i-1} r_{t+j}-\rho^*\pi_{t+j+1}\right)\mathcal{P}_{t+i}^{(n_r)}\times \mathbb{1}\left\{\sum_{j=0}^{i-1}\delta_{t+j}^{(c)}=0\right\}\right]\\
=\;&\lim_{\mathsf{u}\to+\infty} e^{-i\left(\kappa_0^{(r)}-\rho^*\kappa_0^{(\pi)}\right)+\mathsf{A}_{\mathsf{n_r}}-\kappa^{(r)\prime}w_t}\mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\sum_{j=1}^{i-1}\left(\kappa^{(r)}-\rho^*\kappa^{(\pi)}+\mathsf{u}\mathbf{e}_c\right)' w_{t+j}+\left(\mathsf{B}_{n_r}+\kappa^{(\pi)}\right)' w_{t+1}\right)\right]\\
=\;&\lim_{\mathsf{u}\to+\infty}\exp\left\{-i\left(\kappa_0^{(r)}-\rho^*\kappa_0^{(\pi)}\right)+\mathsf{A}_{\mathsf{n_r}}+\mathcal{A}_i^{\mathbb{Q}}\left(\rho^*\kappa^{(\pi)}-\kappa^{(r)}-\mathsf{u}\mathbf{e}_c,\,\rho^*\kappa^{(\pi)}+\mathsf{B}_{n_r}\right)\right.\\
&+\left.\left[\mathcal{B}_i^{\mathbb{Q}}\left(\rho^*\kappa^{(\pi)}-\kappa^{(r)}-\mathsf{u}\mathbf{e}_c,\,\rho^*\kappa^{(\pi)}+\mathsf{B}_{n_r}\right)-\kappa^{(r)}\right]' w_t\right\}.
\end{aligned}$$

Using the same properties, we obtain:

$$\begin{aligned}
&\mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\sum_{j=0}^{i-1} r_{t+j}-\rho^*\pi_{t+j+1}\right)\mathcal{P}_{t+i}^{(n_r)}\times \mathbb{1}\left\{\sum_{j=0}^{i}\delta_{t+j}^{(c)}=0\right\}\right]\\
=\;&\lim_{\mathsf{u}\to+\infty}\exp\left\{-i\left(\kappa_0^{(r)}-\rho^*\kappa_0^{(\pi)}\right)+\mathsf{A}_{\mathsf{n_r}}+\mathcal{A}_i^{\mathbb{Q}}\left(\rho^*\kappa^{(\pi)}-\kappa^{(r)}-\mathsf{u}\mathbf{e}_c,\,\rho^*\kappa^{(\pi)}+\mathsf{B}_{n_r}-\mathsf{u}\mathbf{e}_c\right)\right.\\
&+\left.\left[\mathcal{B}_i^{\mathbb{Q}}\left(\rho^*\kappa^{(\pi)}-\kappa^{(r)}-\mathsf{u}\mathbf{e}_c,\,\rho^*\kappa^{(\pi)}+\mathsf{B}_{n_r}-\mathsf{u}\mathbf{e}_c\right)-\kappa^{(r)}\right]' w_t\right\},
\end{aligned}$$

$$\begin{aligned}
&\mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\sum_{j=0}^{i-1} r_{t+j}-\pi_{t+j+1}\right)e^{-\delta_{t+i}^{(\ell)}}\times \mathbb{1}\left\{\sum_{j=0}^{i}\delta_{t+j}^{(c)}=0\right\}\mathbb{1}\left\{\sum_{j=0}^{i-1}\delta_{t+j}^{(\ell)}=0\right\}\right]\\
=\;&\lim_{\mathsf{u}\to+\infty}\exp\left\{-i\left(\kappa_0^{(r)}-\kappa_0^{(\pi)}\right)+\mathcal{A}_i^{\mathbb{Q}}\left(\kappa^{(\pi)}-\kappa^{(r)}-\mathsf{u}\mathbf{e}_c-\mathsf{u}\mathbf{e}_\ell,\,\kappa^{(\pi)}-\mathsf{u}\mathbf{e}_c-\mathbf{e}_\ell\right)\right.\\
&+\left.\left[\mathcal{B}_i^{\mathbb{Q}}\left(\kappa^{(\pi)}-\kappa^{(r)}-\mathsf{u}\mathbf{e}_c-\mathsf{u}\mathbf{e}_\ell,\,\kappa^{(\pi)}-\mathsf{u}\mathbf{e}_c-\mathbf{e}_\ell\right)-\kappa^{(r)}\right]' w_t\right\},
\end{aligned}$$

and

$$\begin{aligned}
&\mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\sum_{j=0}^{i-1} r_{t+j}-\pi_{t+j+1}\right)e^{-\delta_{t+i}^{(\ell)}}\times \mathbb{1}\left\{\sum_{j=0}^{i}\delta_{t+j}^{(c)}=0\right\}\mathbb{1}\left\{\sum_{j=0}^{i}\delta_{t+j}^{(\ell)}=0\right\}\right]\\
=\;&\lim_{\mathsf{u}\to+\infty}\exp\left\{-i\left(\kappa_0^{(r)}-\kappa_0^{(\pi)}\right)+\mathcal{A}_i^{\mathbb{Q}}\left(\kappa^{(\pi)}-\kappa^{(r)}-\mathsf{u}\mathbf{e}_c-\mathsf{u}\mathbf{e}_\ell,\,\kappa^{(\pi)}-\mathsf{u}\mathbf{e}_c-\mathsf{u}\mathbf{e}_\ell\right)\right.\\
&+\left.\left[\mathcal{B}_i^{\mathbb{Q}}\left(\kappa^{(\pi)}-\kappa^{(r)}-\mathsf{u}\mathbf{e}_c-\mathsf{u}\mathbf{e}_\ell,\,\kappa^{(\pi)}-\mathsf{u}\mathbf{e}_c-\mathsf{u}\mathbf{e}_\ell\right)-\kappa^{(r)}\right]' w_t\right\},
\end{aligned}$$

which is also the last term when $i = n$. Putting all these terms together, we obtain the result of Equation (23).

A.7 Pricing formulas for CDS spreads

The protection seller and protection buyer values are respectively given by:

$$\mathrm{PS}_t^{(n)} = \sum_{i=1}^{n} \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(-\sum_{j=0}^{i-1} r_{t+j} \right) \left(1 - \mathcal{P}_{t+i}^{(n_r)}\right) \left(\mathbb{1}\left\{ \sum_{j=0}^{i-1} \delta_{t+j}^{(c)} = 0 \right\} - \mathbb{1}\left\{ \sum_{j=0}^{i} \delta_{t+j}^{(c)} = 0 \right\} \right) \right],$$

and

$$\mathrm{PB}_t^{(n)} = \mathcal{S}_t^{(n)} \sum_{i=1}^{n} \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(-\sum_{j=0}^{i-1} r_{t+j} \right) \mathbb{1}\left\{ \sum_{j=0}^{i} \delta_{t+j}^{(c)} = 0 \right\} \right]$$

Applying the same pricing principle as in Appendix A.6, we can easily express the protection buyer value as:

$$\begin{aligned}
& \mathcal{S}_t^{(n)} \sum_{i=1}^{n} \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(-\sum_{j=0}^{i-1} r_{t+j} \right) \mathbb{1}\left\{ \sum_{j=0}^{i} \delta_{t+j}^{(c)} = 0 \right\} \right] \\
= & \lim_{\mathsf{u}\to+\infty} \mathcal{S}_t^{(n)} \sum_{i=1}^{n} e^{-i\kappa_0^{(r)} - \kappa^{(r)\prime} w_t} \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(-\sum_{j=0}^{i-1} \left(\kappa^{(r)} + \mathsf{ue}_c \right)' w_{t+j} - \mathsf{ue}_c' w_{t+i} \right) \right] \\
= & \lim_{\mathsf{u}\to+\infty} \mathcal{S}_t^{(n)} e^{-\kappa^{(r)\prime} w_t} \sum_{i=1}^{n} \exp\left\{ -i\kappa_0^{(r)} + \mathcal{A}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{ue}_c, -\mathsf{ue}_c \right) + \mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{ue}_c, -\mathsf{ue}_c \right)' w_t \right\}
\end{aligned}$$

For the protection seller leg, we can separate the term in $(1 - \mathcal{P}_{t+i}^{(n_r)}$ in two and treat these two terms. The first term,

$$\sum_{i=1}^{n} \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(-\sum_{j=0}^{i-1} r_{t+j} \right) \left(\mathbb{1}\left\{ \sum_{j=0}^{i-1} \delta_{t+j}^{(c)} = 0 \right\} - \mathbb{1}\left\{ \sum_{j=0}^{i} \delta_{t+j}^{(c)} = 0 \right\} \right) \right],$$

would be the price of a nominal treasury with recovery payment of the full face value, forgetting the principal repayment at maturity provided no default has happened (the last term of Equation

(20) is missing). The second term,

$$\sum_{i=1}^{n} \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(-\sum_{j=0}^{i-1} r_{t+j} \right) \mathcal{P}_{t+i}^{(n_r)} \left(\mathbb{1}\left\{ \sum_{j=0}^{i-1} \delta_{t+j}^{(c)} = 0 \right\} - \mathbb{1}\left\{ \sum_{j=0}^{i} \delta_{t+j}^{(c)} = 0 \right\} \right) \right] ,$$

is exactly the first row of Equation (20), so it is the price of a nominal treasury, forgetting the principal repayment at maturity provided no default has happened. In the end, taking the difference between these two terms, it is innocuous to add the discounted value of the last payment in both terms since they are canceling out in the difference. We hence obtain that the protection seller value is the difference between the price of a nominal treasury with recovery payment of \$1 and the price of the standard nominal treasury. The result of Equation (25) is obtained by equation the protection buyer and seller values.

A.8 Gradient computation for measurement equations

We use the extended Kalman filter for estimation, which requires the computation of the gradient of the pricing equations in the factors. Since our pricing equations are closed-form, we have closed-form gradients as well. Since these computations are the result of tedious algebra, we only present the results without justification.

Let us start with riskless yields. Given the formulation of $D_t^{(n)}$ and $D_t^{*(n)}$ of Equation (17), and denoting by $r_t^{(n)} = -n^{-1} \log D_t^{(n)}$ and $r_t^{*(n)} = -n^{-1} \log D_t^{*(n)}$, we trivially have:

$$\begin{aligned} \frac{\partial r_t^{(n)}}{\partial w_t} &= \frac{\kappa^{(r)} - \mathcal{B}_n^{\mathbb{Q}}\left(-\kappa^{(r)}, \mathbf{0}\right)}{n} \\ \frac{\partial r_t^{*(n)}}{\partial w_t} &= \frac{\kappa^{(r)} - \mathcal{B}_n^{\mathbb{Q}}\left(\kappa^{(\pi)} - \kappa^{(r)}, \kappa^{(\pi)}\right)}{n} . \end{aligned}$$

Let us turn now to nominal treasuries and TIPS. Continuously compounded yields of these bonds are respectively denoted by $R_t^{(n)} = -n^{-1} \log B_t^{(n)}$ and $R_t^{*(n)} = -n^{-1} \log B_t^{*(n)}$. It is useful to define the differentials with respect to the price instead of the yield directly. Using the chain rule, we have:

$$\begin{aligned} \frac{\partial R_t^{(n)}}{\partial w_t} &= -\frac{1}{n} \times \frac{\partial B_t^{(n)}}{\partial w_t} \times \frac{1}{B_t^{(n)}} \\ \frac{\partial R_t^{*(n)}}{\partial w_t} &= -\frac{1}{n} \times \frac{\partial B_t^{*(n)}}{\partial w_t} \times \frac{1}{B_t^{*(n)}} . \end{aligned}$$

Let us focus on the differential of the nominal bond price first.

$$
\begin{aligned}
\frac{\partial B_t^{(n)}}{\partial w_t} = & \lim_{\mathsf{u}\to+\infty} e^{\mathsf{A}_{n_r}} \sum_{i=1}^{n} e^{-i\kappa_0^{(r)}} \Bigg[\\
& \exp\left\{\mathcal{A}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, \mathsf{B}_{n_r}\right) + \left[\mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, \mathsf{B}_{n_r}\right) - \kappa^{(r)}\right]' w_t\right\} \times \left[\mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, \mathsf{B}_{n_r}\right) - \kappa^{(r)}\right] \\
- & \exp\left\{\mathcal{A}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, \mathsf{B}_{n_r} - \mathsf{u}\mathbf{e}_c\right) + \left[\mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, \mathsf{B}_{n_r} - \mathsf{u}\mathbf{e}_c\right) - \kappa^{(r)}\right]' w_t\right\} \times \Big[\mathcal{B}_i^{\mathbb{Q}}\Big(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, \mathsf{B}_{n_r} - \\
+ & \exp\left\{-n\kappa_0^{(r)} + \mathcal{A}_n^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, -\mathsf{u}\mathbf{e}_c\right) + \left[\mathcal{B}_n^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, -\mathsf{u}\mathbf{e}_c\right) - \kappa^{(r)}\right]' w_t\right\} \times \Big[\mathcal{B}_n^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, -\mathsf{u}\mathbf{e}_c\right)
\end{aligned}
$$

For TIPS, applying a similar principle:

$$
\begin{aligned}
\frac{\partial B_t^{*(n)}}{\partial w_t} = & \lim_{\mathsf{u}\to+\infty} \quad e^{\mathsf{A}_{n_r}} \sum_{i=1}^{n} e^{-i\left(\kappa_0^{(r)} - \rho^*\kappa_0^{(\pi)}\right)} \Bigg[\exp\Big\{\mathcal{A}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c + \rho^*\kappa^{(\pi)},\, \mathsf{B}_{n_r} + \rho^*\kappa^{(\pi)}\right) \\
& + \left[\mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c + \rho^*\kappa^{(\pi)},\, \mathsf{B}_{n_r} + \rho^*\kappa^{(\pi)}\right) - \kappa^{(r)}\right]' w_t\Big\} \times \left[\mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c + \rho^*\kappa^{(\pi)},\, \mathsf{B}_{n_r} + \rho^*\kappa^{(\pi)}\right) - \kappa^{(r)}\right] \\
- & \exp\Big\{\mathcal{A}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c + \rho^*\kappa^{(\pi)},\, \mathsf{B}_{n_r} - \mathsf{u}\mathbf{e}_c + \rho^*\kappa^{(\pi)}\right) \\
& + \left[\mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c + \rho^*\kappa^{(\pi)},\, \mathsf{B}_{n_r} - \mathsf{u}\mathbf{e}_c + \rho^*\kappa^{(\pi)}\right) - \kappa^{(r)}\right]' w_t\Big\} \times \Big[\mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c + \rho^*\kappa^{(\pi)},\, \mathsf{B}_{n_r} - \mathsf{u}\mathbf{e}_c + \rho^*\kappa^{(\pi)}\right) - \\
+ & \sum_{i=1}^{n} e^{-i\left(\kappa_0^{(r)} - \kappa_0^{(\pi)}\right)} \Bigg[\exp\Big\{\mathcal{A}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c - \mathsf{u}\mathbf{e}_\ell + \kappa^{(\pi)},\, -\mathsf{u}\mathbf{e}_c - \mathbf{e}_\ell + \kappa^{(\pi)}\right) \\
& + \left[\mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c - \mathsf{u}\mathbf{e}_\ell + \kappa^{(\pi)},\, -\mathsf{u}\mathbf{e}_c - \mathbf{e}_\ell + \kappa^{(\pi)}\right) - \kappa^{(r)}\right]' w_t\Big\} \times \Big[\mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c - \mathsf{u}\mathbf{e}_\ell + \kappa^{(\pi)},\, -\mathsf{u}\mathbf{e}_c - \mathbf{e}_\ell + \kappa^{(\pi)}\right) \\
- & \exp\Big\{\mathcal{A}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c - \mathsf{u}\mathbf{e}_\ell + \kappa^{(\pi)},\, -\mathsf{u}\mathbf{e}_c - \mathsf{u}\mathbf{e}_\ell + \kappa^{(\pi)}\right) \\
& + \left[\mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c - \mathsf{u}\mathbf{e}_\ell + \kappa^{(\pi)},\, -\mathsf{u}\mathbf{e}_c - \mathsf{u}\mathbf{e}_\ell + \kappa^{(\pi)}\right) - \kappa^{(r)}\right]' w_t\Big\} \times \Big[\mathcal{B}_i^{\mathbb{Q}}\Big(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c - \mathsf{u}\mathbf{e}_\ell + \kappa^{(\pi)},\, -\mathsf{u}\mathbf{e}_c - \mathsf{u}\mathbf{e}_\ell + \kappa^{(\pi} \\
+ & \exp\Big\{-n\left(\kappa_0^{(r)} - \kappa_0^{(\pi)}\right) + \mathcal{A}_n^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c - \mathsf{u}\mathbf{e}_\ell + \kappa^{(\pi)},\, -\mathsf{u}\mathbf{e}_c - \mathsf{u}\mathbf{e}_\ell + \kappa^{(\pi)}\right) \\
& + \left[\mathcal{B}_n^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c - \mathsf{u}\mathbf{e}_\ell + \kappa^{(\pi)},\, -\mathsf{u}\mathbf{e}_c - \mathsf{u}\mathbf{e}_\ell + \kappa^{(\pi)}\right) - \kappa^{(r)}\right]' w_t\Big\} \times \Big[\mathcal{B}_n^{\mathbb{Q}}\Big(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c - \mathsf{u}\mathbf{e}_\ell + \kappa^{(\pi)},\, -\mathsf{u}\mathbf{e}_c - \mathsf{u}\mathbf{e}_\ell + \kappa^{(\pi}
\end{aligned}
$$

Last, for the CDS, let us denote by:

$$
\mathcal{S}_t^{(n)} = \frac{f_n(w_t)}{g_n(w_t)}\,,
$$

where $f_n(\bullet)$ and $g_n(\bullet)$ are explicit functions given by Equation (25). Using differentiation rules:

$$\frac{\partial\, \mathcal{S}_t^{(n)}}{\partial\, w_t} = \frac{1}{g_n(w_t)} \times \frac{\partial\, f_n(w_t)}{\partial\, w_t} - \frac{f_n(w_t)}{g_n(w_t)^2} \times \frac{\partial\, g_n(w_t)}{\partial\, w_t}\,.$$

The differential of $f_n(w_t)$ is easily obtained as a function of the differential of nominal treasuries:

$$\begin{aligned}
&\frac{\partial\, f_n(w_t)}{\partial\, w_t} = \lim_{\mathsf{u}\to+\infty} \sum_{i=1}^{n} e^{-i\kappa_0^{(r)}} \Bigg[\\
&\exp\left\{\mathcal{A}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, \mathbf{0}\right) + \left[\mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, \mathbf{0}\right) - \kappa^{(r)}\right]' w_t\right\} \times \left[\mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, \mathbf{0}\right) - \kappa^{(r)}\right] \\
- \;&\exp\left\{\mathcal{A}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, -\mathsf{u}\mathbf{e}_c\right) + \left[\mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, -\mathsf{u}\mathbf{e}_c\right) - \kappa^{(r)}\right]' w_t\right\} \times \left[\mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, -\mathsf{u}\mathbf{e}_c\right) - \kappa^{(r)}\right] \\
- \;&\exp\left\{\mathsf{A}_{n_r} + \mathcal{A}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, \mathsf{B}_{n_r}\right) + \left[\mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, \mathsf{B}_{n_r}\right) - \kappa^{(r)}\right]' w_t\right\} \times \left[\mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, \mathsf{B}_{n_r}\right) - \kappa^{(r)}\right] \\
+ \;&\exp\left\{\mathsf{A}_{n_r} + \mathcal{A}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, \mathsf{B}_{n_r} - \mathsf{u}\mathbf{e}_c\right) + \left[\mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, \mathsf{B}_{n_r} - \mathsf{u}\mathbf{e}_c\right) - \kappa^{(r)}\right]' w_t\right\} \\
&\quad \times \quad \left[\mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, \mathsf{B}_{n_r} - \mathsf{u}\mathbf{e}_c\right) - \kappa^{(r)}\right]\Bigg],
\end{aligned}$$

and, for the function $g_n(w_t)$:

$$\begin{aligned}
\frac{\partial\, g_n(w_t)}{\partial\, w_t} \;&=\; \lim_{\mathsf{u}\to+\infty} \sum_{i=1}^{n} \Bigg(\exp\left\{-i\kappa_0^{(r)} + \mathcal{A}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, -\mathsf{u}\mathbf{e}_c\right) + \left[\mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, -\mathsf{u}\mathbf{e}_c\right) - \kappa^{(r)}\right]' w_t\right\} \\
&\qquad \times \quad \left[\mathcal{B}_i^{\mathbb{Q}}\left(-\kappa^{(r)} - \mathsf{u}\mathbf{e}_c,\, -\mathsf{u}\mathbf{e}_c\right) - \kappa^{(r)}\right]\Bigg).
\end{aligned}$$

A.9 Supplementary Tables

Table A1: **Cheapest-to-Deliver**

Table A1 shows the results from a panel regression of ILS-BEI on US CDS spreads and various controls using daily observations. Information on the prevailing cheapest-to-deliver outstanding U.S. government nominal bond on each trading day is added as a control as set forth in Klingler and Lando (2018). The sample period is from January 2008 to October 2015. $ILS - BEI$ is the difference in the inflation swap rate and the breakeven inflation rate (Treasury-TIPS) for 2-, 3-. 5-, 7-, and 10-year tenors. $US\ CDS$ spreads are for the 5-year tenor. $HPWNoise$ follows Hu, Pan and Wang (2013). $TIPS\ Noise$ measures average daily deviations in the real yield curve. $LIBOR - OIS$ is the difference in the London Inter-bank Offered Rate and the overnight indexed swap rate. $OTR\ Difference$ is the difference in 10-year Treasury par yield from Gurkaynak, Sack and Wright (2006) less the on-the-run 10-year Treasury yield from Bloomberg. VIX denotes the CBOE Volatility Index. Standard errors are reported in parentheses.

Dep Var: ILS-BEI Spread	(1)	(2)	(3)
US CDS	0.184***	0.217***	0.217***
	(0.050)	(0.052)	(0.052)
ILS-BEI$_{t-1}$	0.822***	0.823***	0.817***
	(0.005)	(0.005)	(0.005)
CtD	−0.134	−0.222**	−0.221**
	(0.095)	(0.099)	(0.099)
HPW Noise		0.634**	0.630**
		(0.291)	(0.290)
TIPS Noise		−1.042***	−1.036***
		(0.182)	(0.182)
LIBOR-OIS		−3.798**	−3.786**
		(1.925)	(1.922)
OTR Difference		−17.962***	−18.021***
		(5.091)	(5.082)
VIX		−0.049	−0.048
		(0.043)	(0.042)
Week	Yes	Yes	Yes
Tenor	No	No	Yes
Observations	9147	9127	9127
R^2	0.149	0.155	0.159

*,**,*** represent statistical significance at the 10%, 5%, and 1% critical threshold, respectively.

Table A2: **Controlling for Foreign Exchange Risk**

Table A2 shows the results from a panel regression of $ILS - BEI$ on US CDS spreads and specifically controls for Euro-Dollar exchange rate movement using daily observations. The sample period is from January 2008 to October 2015. $US\ CDS$ spreads are for the 5-year tenor. $EURUSD$ denotes the 5-year swap spread of the Euro-Dollar basis swap. *Spot* is the spot exchange rate between the Euro and the Dollar. $HPWNoise$ follows Hu, Pan and Wang (2013). $TIPS\ Noise$ measures average daily deviations in the real yield curve. $LIBOR - OIS$ is the difference in the London Inter-bank Offered Rate and the overnight indexed swap rate. $OTR\ Difference$ is the difference in 10-year Treasury par yield from Gurkaynak, Sack and Wright (2006) less the on-the-run 10-year Treasury yield from Bloomberg. VIX denotes the CBOE Volatility Index. Standard errors are reported in parentheses.

Dep Var: ILS-BEI Spread	(1)	(2)	(3)	(4)
US CDS	0.178***	0.208***	0.213***	0.237***
	(0.051)	(0.050)	(0.052)	(0.052)
ILS-BEI$_{t-1}$	0.816***	0.817***	0.817***	0.817***
	(0.005)	(0.005)	(0.005)	(0.005)
EURUSD	−0.076		−0.091*	
	(0.051)		(0.053)	
Spot		10.352		11.461
		(8.067)		(8.313)
HPW Noise			0.611**	0.649**
			(0.291)	(0.291)
TIPS Noise			−1.027***	−1.044***
			(0.182)	(0.182)
LIBOR-OIS			−3.881**	−4.145**
			(1.921)	(1.926)
OTR Difference			−17.077***	−17.430***
			(5.081)	(5.076)
VIX			−0.045	−0.008
			(0.043)	(0.042)
Week	Yes	Yes	Yes	Yes
Tenor	Yes	Yes	Yes	Yes
Observations	9142	9147	9127	9127
$1 - \mathbb{V}(\varepsilon_t)/\mathbb{V}[\Delta(\text{ILS}_t - \text{BEI}_t)]$	0.152	0.152	0.159	0.158

*,**,*** represent statistical significance at the 10%, 5%, and 1% critical threshold, respectively.

Table A3: **FLL Mispricing - January 2008 to October 2015**

Table A3 shows the results from a panel regression of Fleckenstein, Longstaff and Lustig (2014) Treasury-TIPS pairwise mispricing on US CDS spreads and various controls using daily observations. The sample period is from January 2008 to October 2015. FLL $Mispricing$ is the difference between the price of a Treasury bond and the price of a basket of TIPS, inflation swaps, and Treasury strips. $US\ CDS$ spreads are for the 5-year tenor. TTM denotes time-to-maturity. $HPWNoise$ follows Hu, Pan and Wang (2013). $TIPS\ Noise$ measures average daily deviations in the real yield curve. $LIBOR - OIS$ is the difference in the London Inter-bank Offered Rate and the overnight indexed swap rate. $OTR\ Difference$ is the difference in 10-year Treasury par yield from Gurkaynak, Sack and Wright (2006) less the on-the-run 10-year Treasury yield from Bloomberg. VIX denotes the CBOE Volatility Index. Clustered standard errors by bond-pair are reported in parentheses.

Dep Var: FLL Mispricing	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
US CDS	0.665***	0.666***	0.768***	0.631***	0.686***	0.420**	0.501***	0.501***
	(0.203)	(0.203)	(0.226)	(0.187)	(0.219)	(0.172)	(0.181)	(0.180)
FLL-Mis$_{t-1}$	95.408***	95.408***	95.475***	95.411***	95.476***	95.467***	95.473***	94.340***
	(0.752)	(0.752)	(0.762)	(0.751)	(0.762)	(0.758)	(0.761)	(0.782)
TTM	0.003***	0.003***	0.003***	0.003***	0.003***	0.003***	0.003***	−0.111*
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.058)
HPW Noise		−0.597					−0.527	−0.483
		(0.876)					(0.904)	(0.911)
TIPS Noise			−5.159***				−5.440***	−5.416***
			(0.728)				(0.835)	(0.834)
LIBOR-OIS				22.849			2.477	2.547
				(13.765)			(10.031)	(9.989)
OTR Difference					3.768		16.987	16.112
					(33.450)		(35.297)	(34.943)
VIX						1.186***	1.196***	1.190***
						(0.283)	(0.264)	(0.263)
Week	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Bond Pair	No	No	No	No	No	No	No	Yes
Observations	43156	43156	41466	43156	41505	41667	41357	41357
R^2	0.976	0.976	0.977	0.977	0.977	0.977	0.977	0.977

*,**,*** represent statistical significance at the 10%, 5%, and 1% critical threshold, respectively.

Table A4: **Controlling for Repo Premium**

Table A4 shows the results from a panel regression of $ILS - BEI$ on US CDS spreads and various controls including repo premiums D'Amico and Pancost (2018) using daily observations. The sample period is from January 2009 to October 2015. $ILS - BEI$ is the difference in the inflation swap rate and the breakeven inflation rate (Treasury-TIPS) for 2-, 3-. 5-, 7-, and 10-year tenors. $US\ CDS$ spreads are for the 5-year tenor. $HPWNoise$ follows Hu, Pan and Wang (2013). $TIPS\ Noise$ measures average daily deviations in the real yield curve. $LIBOR - OIS$ is the difference in the London Inter-bank Offered Rate and the overnight indexed swap rate. $OTR\ Difference$ is the difference in 10-year Treasury par yield from Gurkaynak, Sack and Wright (2006) less the on-the-run 10-year Treasury yield from Bloomberg. VIX denotes the CBOE Volatility Index. Standard errors are reported in parentheses.

Dep Var: ILS-BEI	(1)	(2)	(3)
US CDS	0.092**	0.078**	0.078**
	(0.037)	(0.039)	(0.038)
ILS-BEI_{t-1}	0.811***	0.810***	0.796***
	(0.006)	(0.006)	(0.006)
Repo Premium	3.105	5.080	4.827
	(5.185)	(5.356)	(5.328)
HPW Noise		0.358	0.353
		(0.264)	(0.263)
TIPS Noise		−0.566**	−0.554**
		(0.223)	(0.222)
LIBOR-OIS		8.056	8.514
		(5.719)	(5.689)
OTR Difference		−7.915*	−7.957*
		(4.572)	(4.548)
VIX		0.079**	0.080**
		(0.035)	(0.035)
Week	Yes	Yes	Yes
Tenor	No	No	Yes
Observations	7950	7930	7930
$1 - \mathbb{V}(\varepsilon_t)/\mathbb{V}[\Delta(\text{ILS}_t - \text{BEI}_t)]$	0.171	0.173	0.182

*,**,*** represent statistical significance at the 10%, 5%, and 1% critical threshold, respectively.