Discussion of *Time-varying Lower Bound of Interest Rates in Europe*

Paper by Cynthia Wu and Dora Xia

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The paper in a nutshell

- Most major currency zones have experienced ZLB in the last couple of years.
- When rates are low, an extra-kick can be needed.
- Implementation of unconventional monetary policies: LSAPs for instance.
- Outside of the U.S., **negative** deposit rate policies

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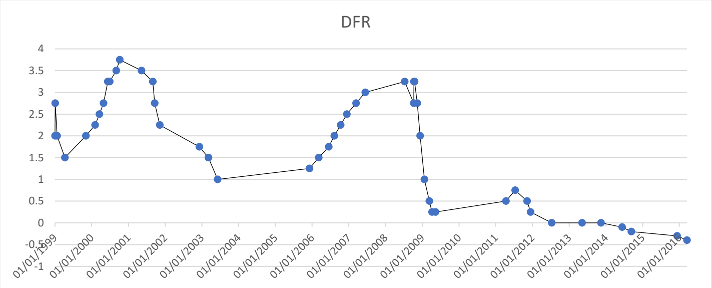
Deposit Facility Rate in the Eurozone

To identification



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Main Question

What are the dynamic effects of NIRP policies on the rest of the yield curve?

The use of a term structure model

- The authors present the puzzle that short-term rates react **exactly 1-for-1** but long-term rates react **more than 1-for-1** with decreases of the deposit facility rate.
- Remember that long-term interest rates $R(t,h)$ are the sum of an expectation and a risk premia component.
- If we expect convergence of short rates (stationarity), then we can assume most of the effect is RP.
- But...

$$r_t = a^* + r_{t-1} + \sigma^* \varepsilon_t, \quad \varepsilon_t \overset{\mathbb{Q}}{\sim} \mathcal{N}(0,1) \tag{1}$$

$$R(t,h) = a_h + r_t \tag{2}$$

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The framework

Term structure factors and *spread factor*

$$\begin{pmatrix} X_t \\ sp_t \end{pmatrix} = \begin{pmatrix} \mu_X \\ \mu_{sp} \end{pmatrix} + \begin{pmatrix} \Phi & 0 \\ 0 & \rho \end{pmatrix} \begin{pmatrix} X_{t-1} \\ sp_{t-1} \end{pmatrix} + \begin{pmatrix} \Sigma^{1/2} & 0 \\ 0 & \sigma \end{pmatrix} \varepsilon_t$$

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- If a shadow rate s_t is affine in these factors, then we are in the Wu & Xia (2016) case and approximate closed-form pricing is available.

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Departure from Wu & Xia (2016)

- Instead of assuming $r_t = \max(s_t, \underline{R})$, we now have:

$$r_t = \max(s_t, \underline{r_t^d})\,, \quad \underline{r_t^d} = \mathrm{DFR}_t + sp_t$$

- DFR_t is independent and a Markov chain conditionally on another Markov chain Δ_t
- $\Delta_t = \pm 1$ and measures monetary policy **direction**.

$$\begin{pmatrix} p & 1-p \\ 1-p & p \end{pmatrix}$$

- the DFR has a bounded support on $\{0, -10, \cdots, -40bps\}$ and conditionally on Δ_t:

$$\mathbb{P}\left(\mathrm{DFR}_t | \Delta_t, \mathrm{DFR}_{t-1}\right)$$
$$= \begin{cases} \pi & \text{for} \quad \mathrm{DFR}_t = \mathrm{DFR}_{t-1} \\ 1-\pi & \text{for} \quad \mathrm{DFR}_t = \mathrm{DFR}_{t-1} + 10 \cdot \Delta_t \end{cases}$$

Findings

- Model fits well and is supported by data (information + likelihood)
- performs better than MS or shadow rate alternatives, so all components are needed (spread + MS + shadow rate)
- Forward looking is needed to explain the reaction of the yield curve to ELB shocks
- Causal effect of -10bps on ELB is about -6 to -8 bps on the 10-y

General Comments

- Paper is going in the right direction to model European bond yields
- The version I have is still at en embryonic stage
- The model is a *natural* extension of Wu & Xia (2016)

Questions

- Is it so natural though?
- Is it the most convenient way to do it?

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Definition

The effective lower bound is the value $\underline{r_t^d}$ such that the *ex-ante* subjective probability of the investors that the short rate goes below $\underline{r_t^d}$ is 0.

$$\mathbb{P}_s(r_{t+k} < \underline{r_t^d} | \mathcal{F}_t) = 0$$

- In term structure models, the subjective and the **true** probability measures are the same so $\mathbb{P} = \mathbb{P}_s$.
- What did investors believe in the early 2000s?
- I would be surprised to see that people believe that the **actual** lower bound was zero.
- You are actually throwing away DFR data before the ZLB episode, why?
- Can you say something about the spread component before the ZLB?

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Comment #2: Puzzle in the paper

The puzzle

long-rates react more than 1-for-1 to decreases in the ELB. *the term premium for the n-period yield is not driven by monetary policy*

- Let us write a simple GATSM.

$$X_t = \mu^* + \begin{pmatrix} 1 & \varphi^* \\ 0 & 1 \end{pmatrix} X_{t-1} + \Sigma^{1/2}\varepsilon_t^*, \quad r_t = \mathbf{1}^T X_t$$

- Loadings are now given by:

$$b_h = \begin{pmatrix} 1 \\ \varphi^* \frac{h+1}{2} \end{pmatrix}$$

- The bigger effect for long-term rates can be reached if lowering the ELB impacts the slope factor.
- Really a puzzle?

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Comment #3: Physical dynamics and risk premia

- Two reasons why dynamics should look like they do: **statistical** and **economic**
- While I understand well the statistical part of it, but not the economics of it.
- Even if the model is nonlinear, it still has a Gaussian flavor to it, so factors should pretty much look like level, slope and curvature.

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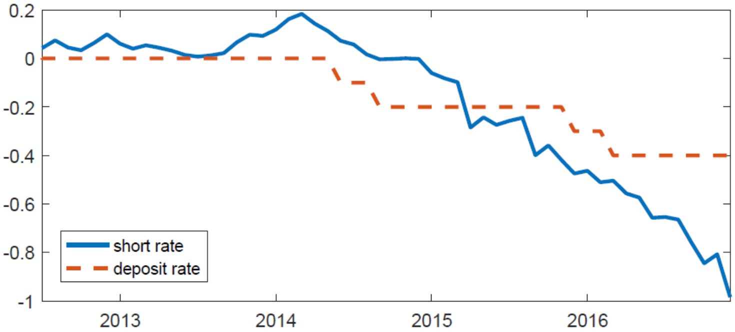
Answer

- Probably not: The spread reflects liquidity effects or excess demand for short-term riskless securities.

$\Longrightarrow$ It is hard to believe that there is no such motive for Long-term securities as well.



Markov Chain dynamics ▸ DFR

- The biggest problem for estimation is the **identification** of the transition probabilities (only $\Delta_t = -1$ is observed in sample)
- The authors circumvent this issue by some critical assumptions.

Assumptions

1. **Symmetric** probabilities π and p
2. Switches in stance of monetary policy stance and DFR are **not priced** (identification of probabilities come directly from prices).

In practice, not likely to hold true.

- If I keep assumption 2, I can easily relax 1.
- Most of the uncertainty of the short-rate now comes from moves in the DFR and this has to be priced (could explain long-run effects).

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Comment #4: Is it the right way to do so?

- Burgeoning literature on term structure models at the lower bound:
  - Christensen (2013), Feunou et al. (2015), Kim and Singleton (2012), Monfort et al. (2017), Roussellet (2016)
- Old and not so old literature on discrete valued processes for term structure models.
  - Piazzesi (2001): dynamics of the target as difference between 2 jump processes
  - Feunou and Fontaine (2011) and Fontaine (2001): discrete choice models with integer valued shocks.
  - Renne (2017): interest rate is given only by Markov chains that are parameterized.
  - Gourieroux et al. (2014): Regime switching models
- Estimation of MS models: specific filters are more well-fitted to your problem than EKF.
- Your model borrows from this literature but does not relate to it directly
- Do you even need the max function in the end?

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An alternative?

- To keep it simple, 2 equations but easily extended to multivariate:

$$\begin{aligned} \ell_t &= \rho^* \ell_{t-1} + \sigma^* \varepsilon_t \\ r_t - \underline{r_t^d} | \mathcal{F}_{t-1} &\sim \Gamma_0 \left[\alpha^* + \beta^* (r_t - \underline{r_t^d}) + \gamma^* (\ell_t - \underline{R})^2, c^* \right] \end{aligned}$$

- Let us call the Gamma variable g_t, then $r_t = \ell_t + g_t$ and closed-form pricing:

$$R(t,h) = a_h + b_h \ell_t + c_h \ell_t^2 + d_h g_t$$

- with exponential-affine pricing kernel, both ELB and interest rate risk are priced and **dynamics are the same with shifted parameters**
- Non-linear IRFs are available **in closed-form**.

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