Discussion of *Slowly Unfolding Disasters*
Paper by Mohammad Ghaderi, Mete Kilic and Sang Byung Seo

Guillaume Roussellet\textsuperscript{1}

\textsuperscript{1}McGill University

HEC-McGill Winter Conference
March 2020
1. **Summary**

2. **Comparing data with simulated process**
   - Simulation exercise
   - Alternative Calibration
   - What is the agent learning about?

3. **Conclusion**

4. **Appendix**
Disaster risk models are successful in explaining Equity premium puzzle, but . . .

- They imply large one-time consumption drops, hence equity price crashes.
- In reality “disasters” correspond to **persistent** decreases in consumption and equity prices.

How to generate this mechanism?

→ Don’t trash your disaster risk model, just introduce some learning.
Disaster risk models are successful in explaining Equity premium puzzle, but . . .

They imply large one-time consumption drops, hence equity price crashes.

In reality “disasters” correspond to persistent decreases in consumption and equity prices.

How to generate this mechanism?

→ Don’t trash your disaster risk model, just introduce some learning.
Disaster risk models are successful in explaining Equity premium puzzle, but . . .

They imply large one-time consumption drops, hence equity price crashes.

In reality “disasters” correspond to **persistent** decreases in consumption and equity prices.

How to generate this mechanism?

→ Don’t trash your disaster risk model, just introduce some learning.
Disaster risk models are successful in explaining Equity premium puzzle, but . . .

They imply large one-time consumption drops, hence equity price crashes.

In reality “disasters” correspond to **persistent** decreases in consumption and equity prices.

How to generate this mechanism?

→ Don’t trash your disaster risk model, just introduce some learning.
Key piece of the model

- **Consumption dynamics:**

  \[
  \Delta c_{t+1} = \mu_c + \sigma_c \varepsilon_{t+1}^c - J_{t+1}. \tag{1}
  \]

- **Jump dynamics (see Appendix):**

  \[
  J_{t+1} | \lambda_t \sim \Gamma_0 (\lambda_t, \mu_\eta). \tag{2}
  \]

- **Gamma intensity dynamics (careful with discrete-time):**

  \[
  \begin{cases}
    \lambda_{t+1} = [s_{t+1} \bar{\lambda}_H + (1 - s_{t+1}) \bar{\lambda}_L] (1 - \rho_\lambda) + \ell_{t+1} \\
    \ell_{t+1} | \lambda_t \sim \Gamma_0 \left( \frac{2\rho_\lambda^2}{\sigma_\lambda^2}, \lambda_t, \frac{\sigma_\lambda^2}{2\rho_\lambda} \right)
  \end{cases} \tag{3}
  \]
This paper is really about dynamics!

- The mechanism is very clear, and pretty plausible:
  - I see the intensity going up.
  - Is it a **persistent change of regime** or just a **transitory** shock?
  - I have to wait until I can be sure.

- Staggered reaction of stock prices (and variance swaps) because of learning.

- Consumption jumps are **smaller** but **more frequent**!

**Main point of my discussion**

→ There’s a revival of identification through consumption only!

→ Let’s look at consumption dynamics in this model.
This paper is really about dynamics!

- The mechanism is very clear, and pretty plausible:
  - I see the intensity going up.
  - Is it a **persistent change of regime** or just a **transitory** shock?
  - I have to wait until I can be sure.

- Staggered reaction of stock prices (and variance swaps) because of learning.

- Consumption jumps are **smaller** but **more frequent**!

**Main point of my discussion**

→ There’s a revival of identification through consumption only!
→ Let’s look at consumption dynamics in this model.
Contents

1 Summary

2 Comparing data with simulated process
   - Simulation exercise
   - Alternative Calibration
   - What is the agent learning about?

3 Conclusion

4 Appendix
Data and simulation

- **Consumption data:** Quarterly postwar 1947-2013 (267 points).
  - I simulate 10,000 trajectories of length $267 \times 12 = 1,068$ months using the calibration of the model.
  - I only focus on consumption growth (asset prices are complicated).
    - Side-note: *Illustrating the pricing mechanism through CRRA (closed-form) looks like a good idea.*

**What am I looking for?**

- Does the simulated data “looks like” real data?
- If yes, there is a chance to **estimate** the model instead of calibrating. (see e.g. Schwenkler (2018))
Consumption data: Quarterly postwar 1947-2013 (267 points).

I simulate 10,000 trajectories of length $267 \times 12 = 1,068$ months using the calibration of the model.

I only focus on consumption growth (asset prices are complicated).

→ Side-note: Illustrating the pricing mechanism through CRRA (closed-form) looks like a good idea.

What am I looking for?

→ Does the simulated data “looks like” real data?

→ If yes, there is a chance to estimate the model instead of calibrating. (see e.g. Schwenkler (2018))
Simulation exercise

Data and simulation

- Consumption data: Quarterly postwar 1947-2013 (267 points).
- I simulate 10,000 trajectories of length $267 \times 12 = 1,068$ months using the calibration of the model.
- I only focus on consumption growth (asset prices are complicated).
  
  → Side-note: Illustrating the pricing mechanism through CRRA (closed-form) looks like a good idea.

What am I looking for?

- Does the simulated data “looks like” real data?
- If yes, there is a chance to estimate the model instead of calibrating. (see e.g. Schwenkler (2018))
Simulation exercise

Data and simulation

- Consumption data: Quarterly postwar 1947-2013 (267 points).
- I simulate 10,000 trajectories of length $267 \times 12 = 1,068$ months using the calibration of the model.
- I only focus on consumption growth (asset prices are complicated).
  
  → Side-note: Illustrating the pricing mechanism through CRRA (closed-form) looks like a good idea.

What am I looking for?

- Does the simulated data “looks like” real data?
- If yes, there is a chance to **estimate** the model instead of calibrating. (see e.g. Schwenkler (2018))
Figure: Two simulated dynamics: disaster and no disaster
Density comparisons

**Consumption Growth (data)**

**simulated path (1)**

**simulated path (2)**

**QQplot**

**QQplot**
Consumption moments

- 18% of trajectories show no disasters (instead of 57% in the paper).

\[
P = \begin{pmatrix} 0.9983 & 0.0017 \\ 0.0208 & 0.9792 \end{pmatrix} \implies \pi = \begin{pmatrix} 0.924 \\ 0.076 \end{pmatrix}
\]

- At least one depression in a year: \(1 - 0.924^{12} = 0.613 >> 7\%\).

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>sd</th>
<th>5%</th>
<th>50%</th>
<th>95%</th>
<th>AC(1)</th>
<th>AC(4)</th>
<th>AC²(1)</th>
<th>AC²(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>data</td>
<td>0.48</td>
<td>0.51</td>
<td>-0.49</td>
<td>0.50</td>
<td>1.22</td>
<td>0.303</td>
<td>0.090</td>
<td>0.213</td>
<td>-0.025</td>
</tr>
<tr>
<td>No dis.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td>0.55</td>
<td>0.87</td>
<td>-2.43</td>
<td>0.89</td>
<td>1.57</td>
<td>-0.092</td>
<td>-0.084</td>
<td>-0.056</td>
<td>-0.052</td>
</tr>
<tr>
<td>50%</td>
<td>0.70</td>
<td>1.13</td>
<td>-1.44</td>
<td>0.96</td>
<td>1.66</td>
<td>-0.015</td>
<td>-0.005</td>
<td>-0.019</td>
<td>-0.015</td>
</tr>
<tr>
<td>95%</td>
<td>0.84</td>
<td>1.38</td>
<td>-0.43</td>
<td>1.02</td>
<td>1.75</td>
<td>0.079</td>
<td>0.101</td>
<td>0.037</td>
<td>0.075</td>
</tr>
<tr>
<td>All</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td>0.04</td>
<td>1.03</td>
<td>-4.50</td>
<td>0.78</td>
<td>1.55</td>
<td>-0.074</td>
<td>-0.071</td>
<td>-0.046</td>
<td>-0.046</td>
</tr>
<tr>
<td>50%</td>
<td>0.53</td>
<td>1.50</td>
<td>-2.38</td>
<td>0.92</td>
<td>1.64</td>
<td>0.019</td>
<td>0.018</td>
<td>-0.011</td>
<td>-0.011</td>
</tr>
<tr>
<td>95%</td>
<td>0.77</td>
<td>2.18</td>
<td>-0.89</td>
<td>1.00</td>
<td>1.74</td>
<td>0.163</td>
<td>0.164</td>
<td>0.136</td>
<td>0.125</td>
</tr>
</tbody>
</table>

Table: Quarterly consumption growth moments (QoQ % change)
Kernel density estimates

**kernel densities (no disasters)**

- **data**
- **simulations**

**kernel densities**

- **data**
- **simulations**

**What should I conclude?**

- Left tail too large, and mode displaced.
- Is this really the dynamics we are looking for?
- What happens if you **estimate** the model?
Comment #2

- Where does the discrepancy in the disaster proportion come from?
  - My gut feeling is that it comes from the calibration, which uses scaling argument of continuous time processes.

What I did

→ I transformed transition probability $p_{0,1} = 0.0001266$ such that the stationary probabilities are 99.4% and 0.6% monthly respectively.

→ Depression probability **per annum**: $1 - 0.994^{12} = 7\%$
Comment #2

- Where does the discrepancy in the disaster proportion come from?
- My gut feeling is that it comes from the calibration, which uses scaling argument of continuous time processes.

What I did

→ I transformed transition probability $p_{0,1} = 0.0001266$ such that the stationary probabilities are 99.4% and 0.6% monthly respectively.

→ Depression probability per annum: $1 - 0.994^{12} = 7%$
Comment #2

- Where does the discrepancy in the disaster proportion come from?
- My gut feeling is that it comes from the calibration, which uses scaling argument of continuous time processes.

What I did

→ I transformed transition probability $p_{0,1} = 0.0001266$ such that the stationary probabilities are 99.4% and 0.6% monthly respectively.

→ Depression probability per annum: $1 - 0.994^{12} = 7%$
How does this alternative calibration impact the pricing moments?
The learning process

- I strongly believe the learning process is useful and that the mechanism at play is plausible.
- This mechanism works as long as consumption is unable to detect the persistence of observed consumption shocks.

Question:
- Is this model the most natural to explore this mechanism?
- Plenty of alternatives, are they relevant?
The learning process

- I strongly believe the learning process is useful and that the mechanism at play is plausible.
- This mechanism works as long as consumption is unable to detect the persistence of observed consumption shocks.

Question:
- Is this model the most natural to explore this mechanism?
- Plenty of alternatives, are they relevant?
I strongly believe the learning process is useful and that the mechanism at play is plausible.

This mechanism works as long as consumption is unable to detect the persistence of observed consumption shocks.

Question:

→ Is this model the most natural to explore this mechanism?

→ Plenty of alternatives, are they relevant?
The learning process

- I strongly believe the learning process is useful and that the mechanism at play is plausible.
- This mechanism works as long as consumption is **unable to detect the persistence** of observed consumption shocks.

**Question:**

→ Is this model the most natural to explore this mechanism?
→ Plenty of alternatives, are they relevant?
Alternatives

- Same model, no regime switching, learning about the intensity process through observed consumption.

- Just regime switches and learning about them (Pakos (2013), David and Veronesi (2013), Johannes et al. (2016)):

  \[ \Delta c_{t+1} = \mu_{s_{t+1}} + \sigma_{s_{t+1}} \varepsilon_{t+1}^c. \]

- Learning about persistence risk? (Andrei et al. (2019))

  \[
  \begin{align*}
  \Delta c_{t+1} &= \mu_c + f_t + \sigma_{c,t+1} \varepsilon_{t+1}^c \\
  f_t &= \theta_t f_{t-1} + \sigma_f \varepsilon_t^f \\
  \theta_t &= \bar{\theta}(1 - \rho_\theta) + \rho_\theta \theta_{t-1} + \sigma_\theta \varepsilon_t^\theta 
  \end{align*}
  \]

- Two (unobserved) components LRR model:

  \[ \Delta c_{t+1} = L_{t+1} + S_{t+1} + \sigma_c \varepsilon_{t+1}^c. \]
Alternatives

- Same model, no regime switching, learning about the intensity process through observed consumption.

- Just regime switches and learning about them (Pakos (2013), David and Veronesi (2013), Johannes et al. (2016)):

  \[ \Delta c_{t+1} = \mu_{s_{t+1}} + \sigma_{s_{t+1}} \varepsilon_{t+1}^c. \]

- Learning about persistence risk? (Andrei et al. (2019))

  \[
  \Delta c_{t+1} = \mu_c + f_t + \sigma_{c,t+1} \varepsilon_{t+1}^c \\
  f_t = \theta_t f_{t-1} + \sigma_f \varepsilon_t^f \\
  \theta_t = \overline{\theta}(1 - \rho_{\theta}) + \rho_{\theta} \theta_{t-1} + \sigma_{\theta} \varepsilon_t^\theta
  \]

- Two (unobserved) components LRR model:

  \[ \Delta c_{t+1} = L_{t+1} + S_{t+1} + \sigma_c \varepsilon_{t+1}^c. \]
Contents

1 Summary

2 Comparing data with simulated process
   - Simulation exercise
   - Alternative Calibration
   - What is the agent learning about?

3 Conclusion

4 Appendix
Conclusion

- Very interesting and well-written paper!
- Learning mechanism is very plausible and works nicely.
- Clarification on calibration and discrete-time formulation.
- Comparison with other asset pricing with learning.
Contents

1 Summary

2 Comparing data with simulated process
   • Simulation exercise
   • Alternative Calibration
   • What is the agent learning about?

3 Conclusion

4 Appendix
**Autoregressive Gamma-zero process**

**Definition**

Let $\lambda_t$ be a non-negative process (almost surely). The process $X_{t+1}$ follows Gamma-zero dynamics with intensity $\lambda_t$ and scale parameter $\mu_x$ if there exists a mixing variable $P_{t+1}$ such that:

$$P_{t+1}|\lambda_t \sim \mathcal{P}(\lambda_t) \quad \text{and} \quad X_{t+1}|P_{t+1} \sim \Gamma_{P_{t+1}}(\mu_x),$$

where $\Gamma_a(b)$ describes the Gamma distribution of density

$$f(x) = \frac{1}{\Gamma(a)b^a} x^{a-1} e^{-x/b}.$$

**Property**

The conditional MGF of $X_{t+1}$ given $\lambda_t$ is given by:

$$\mathbb{E}(uX_{t+1}|\lambda_t) = \exp \left( \frac{u\mu_x}{1 - u\mu_x} \lambda_t \right),$$

where $\mathbb{E}$ denotes the expectation.
Proof that $J_{t+1}$ is a Gamma-zero variable

Let us look for the conditional MGF of $J_{t+1}$.

\[
\mathbb{E} \left( \exp(u J_{t+1}) \bigg| \lambda_t \right) = \mathbb{E} \left[ \exp \left( u \sum_{k=1}^{N_{t+1}} Z_k \right) \bigg| \lambda_t \right] \\
= \mathbb{E} \left[ \exp \left( \sum_{k=1}^{N_{t+1}} \Psi Z(u) \right) \bigg| \lambda_t \right] \quad \text{by indep, } \Psi Z(u) \text{ is CGF.} \\
= \mathbb{E} \left[ \exp \left( N_{t+1} \Psi Z(u) \right) \bigg| \lambda_t \right] \\
= \exp \left[ \lambda_t \left( e^{\Psi Z(u)} - 1 \right) \right] \quad \text{by Poisson prop.}
\]

Since $Z \sim \text{Exp}(\mu_Z)$, then $e^{\Psi Z(u)} = \frac{1}{1/\mu_Z - u} = \frac{1}{1-u\mu_Z}$. Thus:

\[
\mathbb{E} \left( \exp(u J_{t+1}) \bigg| \lambda_t \right) = \exp \left[ \lambda_t \left( e^{\Psi Z(u)} - 1 \right) \right] = \exp \left[ \frac{u \mu_Z}{1 - u \mu_Z} \lambda_t \right] \quad (6)
\]
Proof that our proposed $\lambda_t$ dynamics provides the same moments

- In our proposed dynamics, we have:

$$\begin{cases}
\lambda_{t+1} = \left[ S_{t+1} \bar{\lambda}_H + (1 - S_{t+1}) \bar{\lambda}_L \right] (1 - \rho \lambda) + \ell_{t+1} \\
\ell_{t+1}|\lambda_t \sim \Gamma_0 \left( \frac{2 \rho^2 \lambda}{\sigma^2 \lambda} \cdot \lambda_t, \frac{\sigma^2 \lambda}{2 \rho \lambda} \right)
\end{cases} \quad (7)$$

- It is sufficient to show that $\mathbb{E}_t (\ell_{t+1}) = \rho \lambda \lambda_t$ and $\mathbb{V}_t (\ell_{t+1}) = \sigma^2 \lambda \lambda_t$.

- By the properties of the gamma-zero distribution, we have that $X \sim \Gamma_0(a, b) \Rightarrow \mathbb{E}(X) = ba$ and $\mathbb{V}(X) = 2b^2a$. Thus:

$$\mathbb{E}_t (\ell_{t+1}) = \frac{\sigma^2 \lambda}{2 \rho \lambda} \times \frac{2 \rho^2 \lambda}{\sigma^2 \lambda} \cdot \lambda_t = \rho \lambda \lambda_t$$

$$\mathbb{V}_t (\ell_{t+1}) = 2 \frac{\sigma^2 \lambda}{2 \rho \lambda} \times \rho \lambda \lambda_t = \sigma^2 \lambda \lambda_t$$
Consumption moments

- 56% of trajectories contain disasters.

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>sd</th>
<th>5%</th>
<th>50%</th>
<th>95%</th>
<th>AC(1)</th>
<th>AC(4)</th>
<th>AC²(1)</th>
<th>AC²(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>data</td>
<td>0.48</td>
<td>0.51</td>
<td>-0.49</td>
<td>0.50</td>
<td>1.22</td>
<td>0.303</td>
<td>0.090</td>
<td>0.213</td>
<td>-0.025</td>
</tr>
<tr>
<td>No dis.</td>
<td>5%</td>
<td>0.62</td>
<td>0.81</td>
<td>0.91</td>
<td>1.58</td>
<td>-0.089</td>
<td>-0.085</td>
<td>-0.055</td>
<td>-0.053</td>
</tr>
<tr>
<td></td>
<td>50%</td>
<td>0.75</td>
<td>1.06</td>
<td>0.97</td>
<td>1.66</td>
<td>-0.015</td>
<td>-0.007</td>
<td>-0.016</td>
<td>-0.015</td>
</tr>
<tr>
<td></td>
<td>95%</td>
<td>0.87</td>
<td>1.32</td>
<td>1.03</td>
<td>1.75</td>
<td>0.082</td>
<td>0.102</td>
<td>0.044</td>
<td>0.070</td>
</tr>
<tr>
<td>All</td>
<td>5%</td>
<td>0.49</td>
<td>0.88</td>
<td>0.89</td>
<td>1.57</td>
<td>-0.081</td>
<td>-0.078</td>
<td>-0.047</td>
<td>-0.047</td>
</tr>
<tr>
<td></td>
<td>50%</td>
<td>0.70</td>
<td>1.21</td>
<td>0.96</td>
<td>1.66</td>
<td>-0.003</td>
<td>-0.003</td>
<td>-0.013</td>
<td>-0.013</td>
</tr>
<tr>
<td></td>
<td>95%</td>
<td>0.84</td>
<td>1.67</td>
<td>1.02</td>
<td>1.75</td>
<td>0.122</td>
<td>0.120</td>
<td>0.087</td>
<td>0.093</td>
</tr>
</tbody>
</table>

Table: Quarterly consumption growth moments (QoQ % change)
Kernel density estimates

**kernel densities (no disasters)**

- Data (black)
- Simulations (red)

**kernel densities**

- Data (black)
- Simulations (red)
Bibliography


