Discussion of *Slowly Unfolding Disasters*

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Contents

One-slide summary

- Disaster risk models are successful in explaining Equity premium puzzle, but . . .
- They imply large one-time consumption drops, hence equity price crashes.
- In reality "disasters" correspond to **persistent** decreases in consumption and equity prices.

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Key piece of the model

- Consumption dynamics:

$$\Delta c_{t+1} = \mu_c + \sigma_c \varepsilon^c_{t+1} - J_{t+1} \,. \tag{1}$$

- Jump dynamics (see Appendix):

$$J_{t+1} | \lambda_t \sim \Gamma_0 \left(\lambda_t, \, \mu_\eta \right) \,. \tag{2}$$

- **Gamma** intensity dynamics (careful with discrete-time):

$$\begin{cases} \lambda_{t+1} & = & \left[\mathbf{s_{t+1}} \overline{\lambda}_H + (1 - \mathbf{s_{t+1}}) \overline{\lambda}_L \right] (1 - \rho_\lambda) + \ell_{t+1} \\ \ell_{t+1} | \lambda_t & \sim & \Gamma_0 \left(\frac{2\rho_\lambda^2}{\sigma_\lambda^2} \cdot \lambda_t, \, \frac{\sigma_\lambda^2}{2\rho_\lambda} \right) \end{cases} \tag{3}$$

This paper is really about dynamics!

- The mechanism is very clear, and pretty plausible:
  - I see the intensity going up.
  - Is it a **persistent** change of regime or just a transitory shock?
  - I have to wait until I can be sure.
- Staggered reaction of stock prices (and variance swaps) because of learning.
- Consumption jumps are **smaller** but **more frequent**!

Main point of my discussion

→ There's a revival of identification through consumption only!

→ Let's look at consumption dynamics in this model.

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Data and simulation

- Consumption data: Quarterly postwar 1947-2013 (267 points).
- I simulate 10,000 trajectories of length $267 \times 12 = 1{,}068$ months using the calibration of the model.
- I only focus on consumption growth (asset prices are complicated).
  - → *Side-note: Illustrating the pricing mechanism through CRRA (closed-form) looks like a good idea.*

What am I looking for?

- Does the simulated data "looks like" real data?
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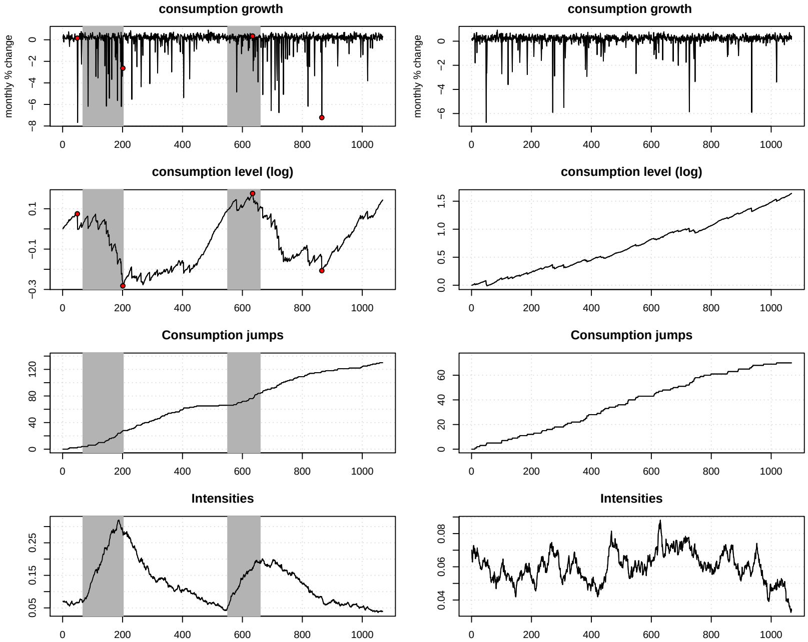
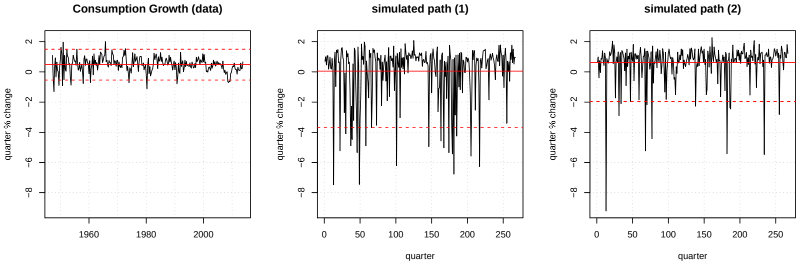
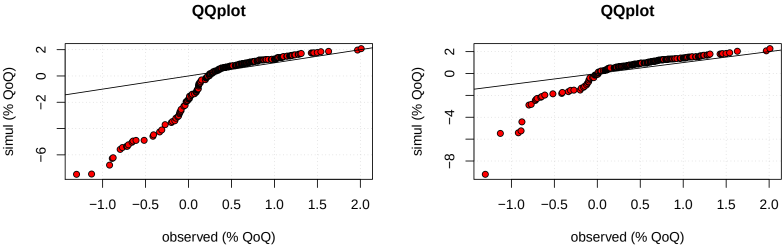
Figure: Two simulated dynamics: disaster and no disaster



Density comparisons





Consumption moments

- 18% of trajectories show no disasters (instead of 57% in the paper).

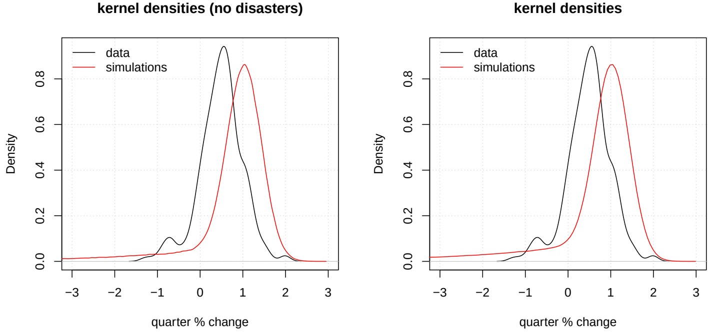
$$P = \begin{pmatrix} 0.9983 & 0.0017 \\ 0.0208 & 0.9792 \end{pmatrix} \implies \pi = \begin{pmatrix} 0.924 \\ 0.076 \end{pmatrix}$$

- At least one depression in a year: $1 - 0.924^{12} = 0.613 >> 7\%$.

		mean	sd	5%	50%	95%	AC(1)	AC(4)	$AC^2(1)$	$AC^2(4)$
	data	0.48	0.51	-0.49	0.50	1.22	0.303	0.090	0.213	-0.025
No dis.	5%	0.55	0.87	-2.43	0.89	1.57	-0.092	-0.084	-0.056	-0.052
	50%	0.70	1.13	-1.44	0.96	1.66	-0.015	-0.005	-0.019	-0.015
	95%	0.84	1.38	-0.43	1.02	1.75	0.079	0.101	0.037	0.075
All	5%	0.04	1.03	-4.50	0.78	1.55	-0.074	-0.071	-0.046	-0.046
	50%	0.53	1.50	-2.38	0.92	1.64	0.019	0.018	-0.011	-0.011
	95%	0.77	2.18	-0.89	1.00	1.74	0.163	0.164	0.136	0.125

Table: Quarterly consumption growth moments (QoQ % change)

Kernel density estimates



What should I conclude?

- Left tail too large, and mode displaced.
- Is this really the dynamics we are looking for?
- What happens if you **estimate** the model?

Comment #2

- Where does the discrepancy in the disaster proportion come from?
- My gut feeling is that it comes from the calibration, which uses scaling argument of continuous time processes.

What I did

→ I transformed transition probability $p_{0,1} = 0.0001266$ such that the stationary probabilities are 99.4% and 0.6% **monthly** respectively.

→ Depression probability **per annum**: $1 - 0.994^{12} = 7\%$

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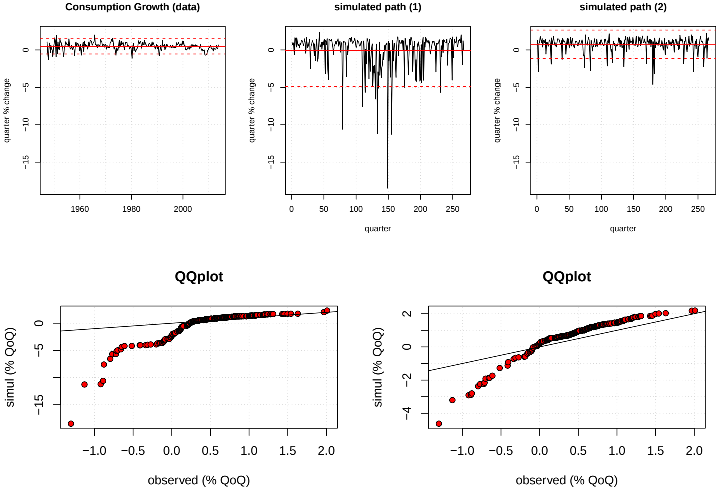
Density comparison

Complete results



- How does this alternative calibration impact the pricing moments?

The learning process

- I strongly believe the learning process is useful and that the mechanism at play is plausible.
- This mechanism works as long as consumption is **unable to detect the persistence** of observed consumption shocks.

Question:

→ Is this model the most natural to explore this mechanism?

→ Plenty of alternatives, are they relevant?

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Alternatives

- Same model, no regime switching, learning about the intensity process through observed consumption.
- Just regime switches and learning about them (Pakos (2013), David and Veronesi (2013), Johannes et al. (2016)): $\Delta c_{t+1} = \mu_{s_{t+1}} + \sigma_{s_{t+1}} \varepsilon^c_{t+1}$.
- Learning about persistence risk? (Andrei et al. (2019))

$$\begin{aligned} \Delta c_{t+1} &= \mu_c + f_t + \sigma_{c,t+1} \varepsilon^c_{t+1} \\ f_t &= \theta_t f_{t-1} + \sigma_f \varepsilon^f_t \\ \theta_t &= \bar{\theta}(1 - \rho_\theta) + \rho_\theta \theta_{t-1} + \sigma_\theta \varepsilon^\theta_t \end{aligned}$$

- Two (unobserved) components LRR model: $\Delta c_{t+1} = L_{t+1} + S_{t+1} + \sigma_c \varepsilon^c_{t+1}$.

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Conclusion

- Very interesting and well-written paper!
- Learning mechanism is very plausible and works nicely.
- Clarification on calibration and discrete-time formulation.
- Comparison with other asset pricing with learning.

Contents

Autoregressive Gamma-zero process

Definition

Let λ_t be a non-negative process (almost surely). The process X_{t+1} follows Gamma-zero dynamics with intensity λ_t and scale parameter μ_x if there exists a mixing variable $\mathcal{P}_{t+1}$ such that:

$$\mathcal{P}_{t+1}|\lambda_t \sim \mathcal{P}\left(\lambda_t\right) \quad \text{and} \quad X_{t+1}|P_{t+1} \sim \Gamma_{\mathcal{P}_{t+1}}(\mu_x)\,, \tag{4}$$

where $\Gamma_a(b)$ describes the Gamma distribution of density $f(x) = \frac{1}{\Gamma(a)b^a} x^{a-1} e^{-x/b}$.

Property

The conditional MGF of X_{t+1} given λ_t is given by:

$$\mathbb{E}\left(uX_{t+1}\middle|\lambda_t\right) = \exp\left(\frac{u\mu_x}{1-u\mu_x}\lambda_t\right) \tag{5}$$

Proof that J_{t+1} is a Gamma-zero variable

Let us look for the conditional MGF of J_{t+1}.

$$\begin{aligned}
\mathbb{E}\left(\exp(uJ_{t+1})\middle|\lambda_t\right) &= \mathbb{E}\left[\exp\left(u\sum_{k=1}^{N_{t+1}} Z_k\right)\middle|\lambda_t\right] \\
&= \mathbb{E}\left[\exp\left(\sum_{k=1}^{N_{t+1}} \Psi_Z(u)\right)\middle|\lambda_t\right] \quad \text{by indep, } \Psi_Z(u) \text{ is CGF.} \\
&= \mathbb{E}\left[\exp\left(N_{t+1}\Psi_Z(u)\right)\middle|\lambda_t\right] \\
&= \exp\left[\lambda_t\left(e^{\Psi_Z(u)} - 1\right)\right] \quad \text{by Poisson prop.}
\end{aligned}$$

Since $Z \sim \mathcal{E}xp\left(\mu_Z\right)$, then $e^{\Psi_Z(u)} = \frac{1/\mu_Z}{1/\mu_Z - u} = \frac{1}{1-u\mu_Z}$. Thus:

$$\mathbb{E}\left(\exp(uJ_{t+1})\middle|\lambda_t\right) = \exp\left[\lambda_t\left(e^{\Psi_Z(u)} - 1\right)\right] = \exp\left[\frac{u\mu_Z}{1-u\mu_Z}\lambda_t\right] \qquad (6)$$

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Proof that our proposed λ_t dynamics provides the same moments

- In our proposed dynamics, we have:

$$\begin{cases} \lambda_{t+1} & = \left[\mathbf{s_{t+1}} \overline{\lambda}_H + (1 - \mathbf{s_{t+1}}) \overline{\lambda}_L \right] (1 - \rho_\lambda) + \ell_{t+1} \\ \ell_{t+1} | \lambda_t & \sim \Gamma_0 \left(\frac{2\rho_\lambda^2}{\sigma_\lambda^2} \cdot \lambda_t, \frac{\sigma_\lambda^2}{2\rho_\lambda} \right) \end{cases} \tag{7}$$

- It is sufficient to show that $\mathbb{E}_t(\ell_{t+1}) = \rho_\lambda \lambda_t$ and $\mathbb{V}_t(\ell_{t+1}) = \sigma_\lambda^2 \lambda_t$.
- By the properties of the gamma-zero distribution, we have that $X \sim \Gamma_0(a, b) \Rightarrow \mathbb{E}(X) = ba$ and $\mathbb{V}(X) = 2b^2 a$. Thus:

$$\mathbb{E}_t(\ell_{t+1}) = \frac{\sigma_\lambda^2}{2\rho_\lambda} \times \frac{2\rho_\lambda^2}{\sigma_\lambda^2} \cdot \lambda_t = \rho_\lambda \lambda_t$$

$$\mathbb{V}_t(\ell_{t+1}) = 2 \frac{\sigma_\lambda^2}{2\rho_\lambda} \times \rho_\lambda \lambda_t = \sigma_\lambda^2 \lambda_t \quad \square$$

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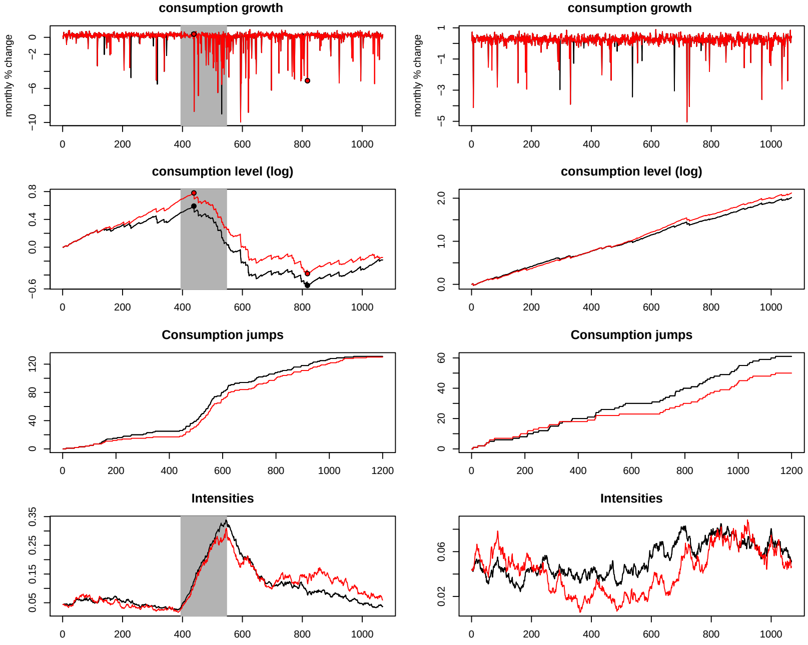

consumption growth
monthly % change
consumption level (log)
Consumption jumps
Intensities
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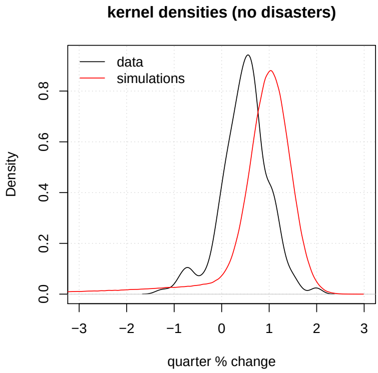

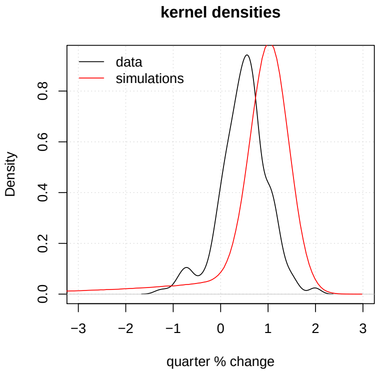

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